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THE OSCILLATING WING WITH AERODYNAMICALLY BALANCED ELEVATOR*

By H. G. Küssner and I. Schwarz

The two-dimensional problem of the oscillating wing with aerodynamically balanced elevator is treated in the manner that the wing is replaced by a plate with bends and stages and the airfoil section by a mean line consisting of one or more straights. The computed formulas and tables permit, on these premises, the prediction of the pressure distribution and of the aerodynamic reactions of oscillating elevators and tabs with any position of elevator hinge in respect to elevator leading edge.

I. INTRODUCTION

The basis of the present report is Küssner's article of the nonstationary lift of airfoils (reference 1), which gives - for the linearized two-dimensional problem of the oscillating wing - general formulas for the calculation of the pressure distribution which are applicable to any periodic form changes of the profile mean line. The example, given in that article (reference 1), dealt with the plate with a single bend, which corresponds to a wing with elevator pivoted in the elevator leading edge. In order to keep the control forces at a minimum, the elevator hinge is, however, usually shifted back or the elevator trailing edge is fitted with a tab. Since the knowledge of the aerodynamic reactions of oscillating wings with such so-called aerodynamically balanced elevators is important for the prediction of the critical speed of wing flutter, the derivation of practical formulas for this case also seemed desirable.

Extension of the theory to include a wing with elevator and tab is comparatively simple. It involves merely the calculation of a plate with two bends instead of one. This problem has, meanwhile, been attacked by F. Dietze

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*"Der schwingende Flügel mit aerodynamisch ausgeglichenem Ruder." Luftfahrtforschung, vol. 17, no. 11/12, Dec. 10, 1940, 337-54.

(reference 2). Treatment of the elevator with set-back hinge, that is, with so-called aerodynamic internal balance, is more difficult. It involves the problem of two independently oscillating plates, which has never been rigorously explored. However, if the gap between both plates is small and the oscillation amplitudes low, a first approximation may be carried out with the formulas developed for one plate without having to satisfy a second flow-off condition. Such a case arises on the double wing. On the more frequently employed types of elevator installations with blunt stabilizer end, no air flows through the gap at small elevator angle. This arrangement is therefore better replaced by a profile mean line passing through between stabilizer trailing edge and elevator leading edge.

If the vertical translatory motion of the elevator is looked upon as a new degree of freedom "stage oscillation," it is possible, in combination with the degrees of freedom of the simple and doubly bent plate, to compute elevator systems with any position of elevator or tab axis. The necessary formulas are evolved in the following, while a new integral representation is employed for computing the pressure distribution.

II. PRESSURE DISTRIBUTION OF THE OSCILLATING WING

The motion of the wing is analyzed first. A uniform rectilinear motion of the wing as a whole with flying speed v is superimposed by a harmonic oscillating motion $z(t)$ with small amplitude at right angles to the direction of flight. To make the problem amenable to mathematical treatment, a linearization of the formulas is first necessary. The airfoil section must be replaced by an aerodynamically equivalent mean line. In the rest position the mean line is to form a straight line coincident with the horizontal x axis, extending from $x = -1$ to $x = +1$. The harmonic oscillation motion of any point of the mean line is then given in complex form by

$$z = f(x)\ e^{i\nu t} \tag{1}$$

with ν = natural frequency, t = time. Ordinarily $f(x)$ is a complex function. Physical significance is to attach to the pure, imaginary term of the equations; hence the mean line is not only subject to translation and rota-

tion but also to any deformations during the oscillation, provided, however, that the oscillation motion is at right angles to the x axis and that the amplitudes are small.

The term "downwash" denotes the vertical velocity of a fluid particle in a coordinate system in which the fluid rests at infinity. The downwash on the mean line is readily given. It consists of a stationary portion due to gliding of the fluid particle past the mean line sloped confommably to $\partial z/\partial x$ at velocity v and of a nonstationary portion due to the transport of the fluid particle with the oscillating mean line conformably to its vertical velocity $\partial z/\partial t$. The assumption that the fluid particle glides at every point along the mean line with constant v is, of course, simply an approximate assumption permissible within the frame of linearization. Small interference velocities Δv relative to flying speed v are ignored. With the reduced frequency $\omega = \frac{i\nu l}{v}$, where l = half the wing chord, equation (1) gives the downwash on the mean line at

$$w = v \frac{\partial z}{\partial x} + \frac{\partial z}{\partial t}$$

$$= v \left(\frac{\partial z}{\partial x} + \frac{\omega z}{l}\right) \tag{2}$$

It is now advisable to introduce a new variable $x = -\cos \theta$ and to consider w as a function of θ and t. Then the Fourier expansion

$$w(\theta,t) = v\ e^{i\nu t}[P_o + 2 \sum_1^\infty P_n \cos n\ \theta] \tag{3}$$

is obtained.

With the downwash given in this form, the two-dimensional airfoil theory yields a definite relation between downwash and pressure distribution on the mean line, whereby pressure means the pressure difference between the upper and the lower sides of the mean line at a certain point x. Next, it is clear that the pressure must also be a harmonic time function if the downwash is. In addition, the pressure must be proportional to the dynamic pressure, that is, proportional to ρV^2. Accordingly,

$$\Pi(\theta,t) = \rho v^2 e^{i\nu t} \left[2\, a_0 \cot\text{an}\, \frac{\theta}{2} + 4 \sum_1^\infty a_n \frac{\sin n\theta}{n} \right] \tag{4}$$

which insures the smooth flow-off at the wing trailing edge. The pressure is tied in with the distribution of bound vortices γ employed in the report (reference 1) through the relation $\Pi = \rho v \gamma$. Then the use of the vortex concept or else Prandtl's acceleration potential affords the general relation between coefficients a_n and P_n:

$$\left. \begin{aligned} a_0 &= \frac{1+T}{2} (P_0 - P_1) + P_1 \\ a_n &= \frac{\omega}{2} P_{n-1} - n P_n - \frac{\omega}{2} P_{n+1}; \quad n \geqq 1 \\ T &= T(-i\omega) = \frac{H_0^{(2)}(-i\,\omega) + i\, H_1^{(2)}(-i\,\omega)}{-H_0^{(2)}(-i\,\omega) + i\, H_1^{(2)}(-i\,\omega)} \end{aligned} \right\} \tag{5}$$

Evaluation of this general result in special cases requires

1. Determination of P_n from the plate motion according to equations (2) and (3);

2. Calculation of a_n from equation (5);

3. Calculation of the pressure distribution equation (4) by summation of the series appearing therein.

To establish relationship between w and Π, the circuitous method of Fourier expansions (3) and (4) is not necessary in the face of the integral relation evolved hereinafter between Π and w. The method used here agrees with the line of reasoning for Poisson's integral. To this end there is put

$$w(\theta,t) = v\, e^{i\nu t}\, W(\theta)$$

$$W(\theta) = P_0 + 2 \sum_1^\infty P_n \cos n\theta$$

and this function is analyzed for $0 \leqq \theta \leqq 2\pi$ rather than for $0 = \theta = \pi$ only, as heretofore.

Let $W(\theta)$ be partly continuous. The Fourier coefficients P_n are

$$P_n = \frac{1}{\pi} \int_0^{\pi} W(\vartheta) \cos n \vartheta \, d \vartheta$$

$$= \frac{1}{2\pi} \int_0^{2\pi} W(\vartheta) \, e^{-in\vartheta} \, d \vartheta, \quad n \geqq 0$$

Then

$$f(z) = \frac{1}{2\pi} \int_0^{2\pi} W(\vartheta) \frac{e^{i\vartheta} + z}{e^{i\vartheta} - z} \, d\vartheta$$

analytically for $|z| < 1$ and is represented by the power expansion (reference 4)

$$f(z) = P_0 + 2 \sum_1^{\infty} P_n \, z^n$$

Multiplication of $a_n (n \geq 1)$ according to (5) by $2z^{n-1}$ and summation of $n = 1$ to ∞ affords

$$2 \sum_1^{\infty} a_n z^{n-1} = \frac{\omega}{2} \left(f(z) + P_0\right) - f'(z) - \frac{\omega}{2} \frac{1}{z^2}\left(f(z) - P_0 - 2P_1 z\right)$$

$$= \frac{\omega}{2} \left[f(z) \left(1 - \frac{1}{z^2}\right) + P_0 \left(1 + \frac{1}{z^2}\right) + \frac{2 P_1}{z} \right] - f'(z)$$

and integration from 0 to z gives

$$2 \sum_1^{\infty} a_n \frac{z^n}{n} = \frac{\omega}{2} \int_0^{z} \left[f(z) \left(1 - \frac{1}{z^2}\right) + P_0 \left(1 + \frac{1}{z^2}\right) + \frac{2 P_1}{z} \right] dz - f(z) + P_0$$

Insertion of the foregoing integral representations for $f(z)$, P_0, P_1, followed by exchange of the integrations, yields

$$2 \sum_1^\infty a_n \frac{z^n}{n} = -\frac{1}{\pi}\int_0^{2\pi} W(\vartheta)\left[\omega\, i \ln(1-e^{-i\vartheta}z)\sin\vartheta + \frac{z}{e^{i\vartheta}-z}\right]d\vartheta$$

With $z \rightarrow e^{i\Theta}$, it affords, if the left side converges,

$$2 \sum_1^\infty a_n \frac{e^{in\Theta}}{n} = -\frac{1}{\pi}\int_0^{2\pi} W(\vartheta)\left[\omega\, i \ln(1-e^{i(\Theta-\vartheta)})\sin\vartheta + \frac{e^{i\Theta}}{e^{i\vartheta}-e^{i\Theta}}\right]d\vartheta - W(\Theta)$$

The integral on the right side, at least the integral over the portion with $\dfrac{e^{i\Theta}}{e^{i\vartheta}-e^{i\Theta}}$, is to be taken as Cauchy's principal value, at which the critical point of the integrand, $\vartheta = \Theta$, is approached symmetrically from both sides. The additive term $-W(\Theta)$ follows the exact execution of the limit transition (reference 4).

For abbreviation there is put

$$w\, i \ln(1-e^{i(\Theta-\vartheta)})\sin\vartheta + \frac{e^{i\Theta}}{e^{i\vartheta}-e^{i\Theta}} = K(\vartheta,\Theta)$$

Since $W(\vartheta)$ is precisely and periodically of the period 2π,

$$\int_0^{2\pi} W(\vartheta)K(\vartheta,\Theta)d\vartheta = \int_0^{\pi} W(\vartheta)\,[K(\vartheta,\Theta) + K(-\vartheta,\Theta)]\,d\vartheta$$

and consequently

$$4\, i \sum_1^\infty a_n \frac{\sin n\Theta}{n} = -\frac{1}{\pi}\int_0^{\pi} W(\vartheta)K_1(\vartheta,\Theta)d\vartheta$$

with

$$K_1(\vartheta,\Theta) = K(\vartheta,\Theta) + K(-\vartheta,\Theta) - K(\vartheta,-\Theta) - K(-\vartheta,-\Theta)$$

or computed

$$K_1(\vartheta,\Theta) = \omega\ i\ (-2 \sin\vartheta\ L(\Theta,\vartheta)) + \frac{2\ i \sin\Theta}{\cos\vartheta - \cos\Theta}$$

where

$$L(\Theta,\vartheta) = \ln\left|\frac{\sin\frac{\Theta+\vartheta}{2}}{\sin\frac{\Theta-\vartheta}{2}}\right| = \frac{1}{2}\ln\frac{1-\cos(\Theta+\vartheta)}{1-\cos(\Theta-\vartheta)} \qquad (6)$$

and hence

$$4\sum_1^\infty a_n \frac{\sin n\Theta}{n} = \frac{}{\pi}\int_0^\pi W(\vartheta)\left[\omega L(\Theta,\vartheta)\sin\vartheta - \frac{\sin\Theta}{\cos\vartheta - \cos\Theta}\right]d\vartheta$$

Herewith the summation cited under 3 is carried out once for all. Adding the missing terms gives for the pressure distribution

$$\Pi(\Theta,t) = \rho\ v^2\ e^{i\nu t}\left[(1+T)(P_0 - P_1) + 2\ P_1\right]\cot\!an\frac{\Theta}{2}$$

$$+ \rho\ v\ \frac{2}{\pi}\int_0^\pi w(\vartheta,t)\left[\omega\ L(\Theta,\vartheta)\sin\vartheta - \frac{\sin\Theta}{\cos\vartheta - \cos\Theta}\right]d\vartheta \qquad (7)$$

or, after expressing the Fourier coefficients P_0 and P_1 by integrals,

$$\Pi(\Theta,t) = \rho v\frac{2}{\pi}\int_0^\pi w(\vartheta,t)\left[\left\{\frac{1+T}{2}(1-\cos\vartheta)+\cos\vartheta\right\}\cot\!an\frac{\Theta}{2}+\omega L(\Theta,\vartheta)\sin\vartheta - \frac{\sin\Theta}{\cos\vartheta-\cos\Theta}\right]d\vartheta$$

(8)

Π is herewith given by w and, according to (2), by z. In conclusion it is pointed out that an independent deduction of the same result based upon the known solution of a simple integral equation, will be found in Schwarz's report (reference 3). (H. Söhngen also reached this result according to oral communication.)

III. FORCES AND MOMENTS

With pressure distribution Π given in function of Θ, the force exerted on a region $x_1 < x < 1$ of the mean line is

$$K_1 = \int_{x_1}^{1} \Pi \; d\,x = \int_{\chi_1}^{\pi} \Pi \; (\Theta, t) \sin \Theta \; d\,\Theta \qquad (9)$$

where $x_1 = - \cos \chi_1$. The area $x = 1$ and $\Theta = \pi$, respectively, indicates the wing trailing edge, the end of the mean line. The moment about the reference point $x_2 = - \cos \chi_2$ is defined by

$$M_1 = \int_{x_1}^{1} \Pi (x - x_2) \; d\,x =$$

$$= \int_{\chi_1}^{\pi} \Pi (\Theta, t)(\cos \chi_2 - \cos \Theta) \sin \Theta \; d\,\Theta \qquad (10)$$

with the semichord put at $l = 1$. K_1 and M_1 apply solely to a plate strip of width 1. For the force and the moment of any plate with chord $2l$ and span b, it is usual to put

$$K = K_1 \; l b$$

$$M = M_1 \; l^2 b$$

The moment is positive when nose-heavy.

IV. DIVISION OF WING MOTION IN SIX DEGREES OF FREEDOM

The linearization of the problem in question makes it possible to split complicated motions and form changes of the wing into so-called degrees of freedom from which the final solution is afforded by superposition.

The analysis bases on a wing with aerodynamically balanced elevator and tab, having the mean line shown in figure 1. The wing as a whole is free to move. The movability of the elevator is restriced to the extent that one of its points, the elevator axis, is rigidly connected with the wing, while the tab axis itself is rigidly joined with the elevator. Accordingly, the state of motion can be superimposed by the following four portions (fig. 2):

a) Flapping motions,

b) Wing torsion oscillations about the forward neutral point $x = -1/2$,

c), d) Elevator torsion oscillations about the elevator hinge [$x = -\cos \chi_R$; Elevator leading edge at $x = -\cos \varphi$; wing trailing edge at $x = -\cos (\varphi - \delta_R)$],

e), f) Tab torsion oscillations about the tab hinge [$x = -\cos \chi_H$; tab leading edge at $x = -\cos \psi$; elevator trailing edge at $x = -\cos(\psi - \delta_H)$],

The elevator torsional oscillation is divided into two parts and treated separately, namely, as a torsional oscillation about the elevator leading edge and a translatory oscillation, termed "step oscillation" (figs. 3a and 3b). The same applies to the tab. The advantage accruing from this division is that both portions in the main depend only upon one geometrical parameter φ or ψ. The actual elevator oscillation with any hinge and any amplitudes follows then as a linear form of these two portions.

Accordingly, any wing-elevator-tab oscillation can be built up from the following six degrees of freedom:

a) Flapping oscillation of the whole wing, pure translation of wing,

b) Torsional oscillation of wing about the forward neutral point,

c) Torsional oscillation of elevator about its leading edge,

d) Step oscillation of elevator, elevator translation,

e) Torsional oscillation of tab about its leading edge,

f) Step oscillation of tab, translation of tab.

It should be borne in mind that elevator and tab are included in the wing and the tab in the elevator. The amplitudes of these six degrees of freedom are denoted with A, B, C, D, and F. Amplitude D follows from the requirement that the point $x = -\cos\chi_R$ corresponding to the elevator axis remain fixed at the oscillation superimposed from degrees of freedom c and d. Now this point experiences through degree of freedom c the maximum deflection $c(\cos\varphi - \cos\chi_R)$, through degree of freedom d the deflection D. If the elevator axis is to remain fixed, it must be:

$$C(\cos\varphi - \cos\chi_R) + D = 0 \tag{11}$$

The corresponding formula for the elevator tab is:

$$E(\cos\psi - \cos\chi_H) + F = 0 \tag{12}$$

The various degrees of freedom and superpositions are illustrated in figures 2, 3a, and 3b. For d and f two different cases are involved, one the "open stage," allowing free flow through the elevator gap, the other, the "closed stage" where no flow passes the elevator gap. The former is approximately achieved by a double wing with sharp stabilizer tip and thin elevator or a less vertically arranged elevator. The second case is that of arrangements with blunt stabilizer tip and thicker elevator, where the thickness itself prevents any flow through the gap at small elevator angles and even at larger angles throttles the flow considerably (figs. 3a, 3b). The open stage, denoted by $^{\circ}$, consists of two flat, parallel plates. From stationary investigations of bent plates with gap (references 4 and 5) it is known that in the extreme case of infinitely small gap the pressure distribution is unaffected by a vertical translation of the elevator, if the translation is small. Compliance with a second flow-off condition is therefore unnecessary in this extreme case, and the pressure distribution is achieved from the known solution of the integral equation of a plate extending from stabilizer leading edge to elevator trailing edge. It is assumed that this holds for nonstationary flow also.

The closed stage, denoted by $^{-}$, consists of two flat, parallel plates joined by a slope with the projection $2l\tau_{SR}$. The mean line therefore is an uninterrupted straight line. It is assumed that the width $2l\tau_{SR}$ of the gap bridged by a straight line is small and the terms approaching zero with τ_{SR} are omitted in the subsequent formulas. This leaves then only a logarithmic term in the expression for the stationary elevator force, which contains τ_{SR}. By disappearing width $\tau_{SR} \rightarrow 0$, that is, on becoming vertical, this term obviously tends toward ∞, since the linearized theory holds only for small inclination angles of the mean line. This difficulty can be avoided by selecting either a finite width $2l\tau_{SR}$ or prescribing a "break-off." Logarithmic singularities frequently occurring in the results of theoretical physics are usually made harmless by break-off directions.

The stage oscillation of the tab is also divided into "open" and "closed."

In respect to the substitution of the wing profile for a mean line consisting of one or more straights, it may be stated in general that a makeshift is involved. If sufficiently accurate nonstationary pressure measurement were available, a substitute mean line could be computed which reproduces the recorded pressures within the frame of the linearized theory. The mean line would generally be curved and encumbered with a large number of parameters.

V. FORCES AND MOMENTS OF THE DEGREES OF FREEDOM

K lift of total wing

M_0 moment of total wing about the forward neutral point

R lift of elevator

N moment of elevator about its leading edge

P lift of elevator tab

Q moment of ~~elevator~~ tab about its leading edge

These forces and moments $K, \ldots Q$ are composed of various portions produced by the degrees of freedom $a, \ldots f$:

$$K = K_a + K_b + \ldots + K_f$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$Q = Q_a + Q_b + \ldots + Q_f$$

where, for example, K_a denotes the portion of the wing lift obtaining from degree of freedom a with amplitude A. Accordingly, the 36 quantities

$$K_g, M_{og}, R_g, N_g, P_g, Q_g, (g = a, \ldots f)$$

must be explored. They become dimensionless and are freed of the factor $e^{i\nu t}$ through

$$\left.\begin{aligned}
K_g &= \pi \rho v^2 l b G e^{i\nu t} k_g \\
M_{og} &= \pi \rho v^2 l^2 b G e^{i\nu t} m_g \\
R_g &= \pi \rho v^2 l b G e^{i\nu t} r_g \\
N_g &= \pi \rho v^2 l^2 b G e^{i\nu t} n_g \\
P_g &= \pi \rho v^2 l b G e^{i\nu t} p_g \\
Q_g &= \pi \rho v^2 l^2 b G e^{i\nu t} q_g
\end{aligned}\quad g = a, \ldots, f \right\} \tag{13}$$

These are the final formulas, which reduce the forces and moments to the dimensionless forces and moments of the separate degrees of freedom:

$$\left.\begin{aligned}
K &= \pi \rho v^2 l b e^{i\nu t} \Sigma G k_g \\
M_o &= \pi \rho v^2 l^2 b e^{i\nu t} \Sigma G m_g \\
R &= \pi \rho v^2 l b e^{i\nu t} \Sigma G r_g \\
N &= \pi \rho v^2 l^2 b e^{i\nu t} \Sigma G n_g \\
P &= \pi \rho v^2 l b e^{i\nu t} \Sigma G p_g \\
Q &= \pi \rho v^2 l^2 b e^{i\nu t} \Sigma G q_g
\end{aligned}\right\} \tag{14}$$

The summation must be extended over all the occurring degrees of freedom g. G denotes the amplitude of degree of freedom g. The elevator moment N is referred to the elevator leading edge, the tab moment Q to its leading edge. The 36 aerodynamic factors to be defined, should be arranged as shown in figure 4. This comprises 9 fields of four factors each, arranged in 3 rows and 3 columns, respectively. The relationship of ω is not marked. The factors in the fields of the second row and second column depend on φ, those in the third row and third column on ψ (fig. 5).

Hence

1. 4 factors are independent of φ and ψ
2. 12 factors are functions of φ
3. 12 factors are functions of ψ
4. $2 \cdot 4$ factors are functions of φ and ψ

VI. PRESSURE DISTRIBUTION AND AERODYNAMIC COEFFICIENTS OF THE DEGREES OF FREEDOM

a) Flapping Oscillation

The plate oscillates harmonically by shifting parallel at right angles to the direction of flow:

$$z_a = A\ e^{i\nu t}$$

Then

$$w_a = v\ \omega\ A\ e^{i\nu t}$$

the Fourier coefficients of w_a are

$$P_o = \omega\ A,\ P_n = 0 \quad \text{for} \quad n > 0$$

whence the a_n's follow at

$$a_o = \frac{1 + T}{2}\ \omega\ A,\ a_1 = \frac{\omega}{2}\ A,\ a_n = 0 \quad \text{for} \quad n > 1$$

Then:

$$\Pi_a = \Pi_a (A; \Theta, t) =$$

$$= \rho v^2 e^{i\nu t} A\left[(1+T)\omega \cot \frac{\Theta}{2} + \right.$$

$$\left. + 2\omega^2 \sin\Theta\right] \qquad (15)$$

Integration of this pressure distribution according to equations (9) and (10) affords the following forces and moments and their dimensionless coefficients, respectively:

$$\left.\begin{aligned} k_a &= \omega(1+T) + \omega^2 \\ m_a &= \frac{1}{2}\omega^2 \\ \pi r_a(\varphi) &= (1+T)\,\omega\,\Phi_{31} + \omega^2 \Phi_3 \\ \pi n_a(\varphi) &= \frac{1}{2}\omega\,\Phi_8(1+T) + \frac{1}{2}\omega^2\Phi_4 \\ p_a(\psi) &= r_a(\psi) \\ q_a(\psi) &= n_a(\psi) \end{aligned}\right\} \quad \ldots (16)$$

Quantities $\Phi_n = \Phi_n(\varphi)$ are functions of φ and are found in the list of the Φ function (sec. VII, 5). Since quantity φ does not appear in Π_a as parameter, the integrals for p_a, q_a differ from those for r_a, n_a in the ψ substituted for φ.

b) Torsional Oscillation

Restricted to small torsional angles, the plate motion is represented by

$$z_b = B\left(\frac{1}{2} - \cos\Theta\right) e^{i\nu t}.$$

which affords

$$w_b = v B e^{i\nu t}\cdot\left[\omega\left(\frac{1}{2} - \cos\Theta\right) + 1\right],$$

whence the Fourier coefficients

$$P_0 = B\left(1 + \frac{\omega}{2}\right);\ P_1 = -\frac{\omega}{2}B;\ P_n = 0 \text{ for } n > 1$$

The a_n coefficients follow at

$$a_0 = \frac{1+T}{2}(1+\omega)B - \frac{\omega}{2}B$$

$$a_1 = \omega B + \frac{\omega^2}{4}B$$

$$a_2 = -\frac{\omega^2}{4}B,\ a_n = 0 \text{ for } n > 2$$

Then the pressure distribution is:

$$\Pi_b = \Pi_b(B; \Theta, t) =$$
$$= \varrho v^2 e^{i\nu t}\cdot B\cdot\left[\left\{(1+\omega)(1+T) - \omega\right\}\operatorname{cotg}\frac{\Theta}{2} + (4\omega + \omega^2)\sin\Theta - \omega^2\sin\Theta\cos\Theta\right] \quad \ldots\ldots (17)$$

Integration gives the force and moment coefficients at

$$\left.\begin{aligned} k_b &= (1+\omega)(1+T) + \omega + \frac{1}{2}\omega^2 \\ m_b &= \omega + \frac{3}{8}\omega^2 \\ \pi r_b(\varphi) &= (1+T)(1+\omega)\Phi_{31} + \omega\Phi_{32} + \frac{\omega^2}{4}\Phi_6 \\ \pi n_b(\varphi) &= \frac{1}{2}(1+\omega)(1+T)\Phi_8 + \frac{1}{2}\omega\Phi_9 + \frac{1}{4}\omega^2\Phi_7 \\ p_b(\psi) &= r_b(\psi) \\ q_b(\psi) &= n_b(\psi) \end{aligned}\right\} \quad (18)$$

c) Torsional Oscillation of Elevator

Consider the plate with a single bend, the bent portion executing harmonic torsional oscillations about the break. In this instance the state of motion is expressed by

$$z_c = \begin{cases} 0 & \text{for } 0 \leqq \Theta \leqq \varphi \\ C e^{i\nu t}(\cos\varphi - \cos\Theta) & \text{for } \varphi \leqq \Theta \leqq \pi \end{cases}$$

if restricted to small elevator oscillations. For w_c it affords

$$w_c = \begin{cases} 0 & \text{for } 0 \leqq \Theta \leqq \varphi \\ v C e^{i\nu t}\cdot[\omega(\cos\varphi - \cos\Theta) + 1] & \text{for } \varphi \leqq \Theta \leqq \pi. \end{cases}$$

For the calculation of Π_c the integral representation of section II is resorted to. Fourier analysis gives the first two Fourier coefficients of the downwash at

$$P_0 = \frac{C}{\pi}\left[\pi - \varphi + \omega\cdot\left\{(\pi - \varphi)\cos\varphi + \sin\varphi\right\}\right],$$

$$P_1 = \frac{C}{\pi}\left[-\sin\varphi + \frac{\omega}{2}\left\{-\pi + \varphi - \sin\varphi\cdot\cos\varphi\right\}\right].$$

whence the factor of $\cot\frac{\Theta}{2}$ at

$$(P_0 - P_1)(1+T) + 2P_1$$
$$= \frac{C}{\pi}\left[(1+T)\Phi_1 + (1+T)\,\omega\,\frac{1}{2}\Phi_2 - 2\sin\varphi - \omega\Phi_3\right]$$

On separation according to the 5 functions of ω, namely,

$$(1+T),\ (1+T)\,\omega,\ 1,\ \omega,\ \omega^2$$

the integral representation changes to

$$\Pi_c = \Pi_c(C, \varphi; \Theta, t) = \varrho v^2 e^{i\nu t}\cdot\frac{C}{\pi}$$
$$\times\left[(1+T)\Pi_{c1} + (1+T)\,\omega\,\Pi_{c2} + \Pi_{c3} + \omega\Pi_{c4} + \omega^2\Pi_{c5}\right] \quad \ldots\ldots (19)$$

with

$$\Pi_{c1} = \Phi_1 \operatorname{cotg}\frac{\Theta}{2}$$

$$\Pi_{c2} = \frac{1}{2}\Phi_2 \operatorname{cotg}\frac{\Theta}{2}$$

$$\Pi_{c3} = -2\sin\varphi\cdot\operatorname{cotg}\frac{\Theta}{2} - 2\int_{\varphi}^{\pi}\frac{\sin\Theta\cdot d\vartheta}{\cos\vartheta-\cos\Theta}$$

$$\Pi_{c4} = -\Phi_3\operatorname{cotg}\frac{\Theta}{2} + \\ + 2\int_{\varphi}^{\pi}\left[\frac{\cos\vartheta-\cos\varphi}{\cos\vartheta-\cos\Theta}\sin\Theta + L(\Theta,\vartheta)\right]\sin\vartheta\, d\vartheta$$

$$\Pi_{c5} = 2\int_{\varphi}^{\pi}(\cos\varphi-\cos\vartheta)\cdot L(\Theta,\vartheta)\sin\vartheta\, d\vartheta .$$

The integrations give

$$\Pi_{c3} = -2\sin\varphi\cdot\operatorname{cotg}\frac{\Theta}{2} + 2L(\Theta,\varphi)$$

$$\Pi_{c4} = -\Phi_3\operatorname{cotg}\frac{\Theta}{2} + 4(\pi-\varphi)\sin\Theta \\ + 4(\cos\varphi-\cos\Theta)\,L(\varphi,\Theta)$$

$$\Pi_{c5} = [(\pi-\varphi)\cdot 2\cos\varphi + \sin\varphi]\sin\Theta \\ -(\pi-\varphi)\sin\Theta\cos\Theta + L(\varphi,\Theta)(\cos\varphi-\cos\Theta)^2 .$$

The same result is achieved by direct summation of the Fourier series by a different method (reference 1), which, however, involves much more paper work and is therefore omitted.

Integration of the pressure distribution (19) by equations (9) and (10) affords the quantities k_c, m_c, r_c, n_c at:

$$\left.\begin{aligned}
\pi k_c(\varphi) &= (1+T)\left(\Phi_1+\frac{1}{2}\omega\Phi_2\right)+\omega\Phi_3+\frac{1}{2}\omega^2\Phi_4\\
\pi m_c(\varphi) &= \Phi_5+\frac{1}{2}\omega\Phi_6+\frac{1}{4}\omega^2\Phi_7\\
\pi^2 r_c(\varphi) &= (1+T)\left(\Phi_1+\frac{1}{2}\omega\Phi_2\right)\Phi_{31}+\Phi_{35}+\omega\Phi_{36}\\
&\qquad +\frac{1}{2}\omega^2\Phi_{37}\\
\pi^2 n_c(\varphi) &= (1+T)\left(\Phi_1+\frac{1}{2}\omega\Phi_2\right)\frac{1}{2}\Phi_8+\Phi_{10}+\frac{1}{2}\omega\Phi_{11}\\
&\qquad +\frac{1}{4}\omega^2\Phi_{12}
\end{aligned}\right\}\quad(20)$$

While these quantities depend on φ only, the force and moment coefficients of the elevator tab involve both φ and ψ in p_c and q_c. The result is:

$$\left.\begin{aligned}
\pi^2 p_c(\varphi,\psi) &= (1+T)X_1+(1+T)\omega X_2\\
&\qquad +X_3+\omega X_4+\omega^2 X_5\\
\pi^2 q_c(\varphi,\psi) &= (1+T)X_6+(1+T)\omega X_7\\
&\qquad +X_8+\omega X_9+\omega^2 X_{10}
\end{aligned}\right\}\quad(21)$$

$X_i = X_i(\varphi,\psi)$ being functions of φ and ψ, given in the list of the X function in section VII, 6.

d) Stage Oscillation of Elevator

$\mathring{d}$. Open stage.

In degree of freedom d two cases are involved: the open stage and the closed stage. The first defines the deformation

$$\begin{aligned}
z_{\mathring{d}} = z_{\mathring{d}}^{\circ}(D,\varphi,\Theta) &= 0 && \text{for } 0 \leqq \Theta < \varphi\\
&= D e^{i\nu t} && \text{for } \varphi < \Theta \leqq \pi .
\end{aligned}$$

which affords

$$\begin{aligned}
w_{\mathring{d}}^{\circ} &= 0 && \text{for } 0 \leqq \Theta < \varphi\\
&= \nu D e^{i\nu t}\omega && \text{for } \varphi < \Theta \leqq \pi .
\end{aligned}$$

For computing the pressure distribution, equation (7) is again resorted to. The first Fourier coefficients of the downwash are

$$P_0 = \frac{D\omega}{\pi}(\pi-\varphi),\quad P_1 = -\frac{D\omega}{\pi}\sin\varphi .$$

giving the factor of cotan $\frac{\theta}{2}$ at

$$(P_0-P_1)(1+T)+2P_1 = \frac{D}{\pi}\cdot[-2\omega\sin\varphi+(1+T)\omega\Phi_1].$$

Then the pressure distribution is according to (7):

$$\Pi_{\mathring{d}}^{\circ} = \Pi_{\mathring{d}}^{\circ}(D,\varphi;\ \Theta,t) = \varrho v^2 e^{i\nu t}\cdot\frac{D}{\pi} \\ \times[(1+T)\Pi_{\mathring{d}1}+(1+T)\omega\Pi_{\mathring{d}2}^{\circ}+\Pi_{\mathring{d}3}^{\circ}+\omega\Pi_{\mathring{d}4}^{\circ}+\omega^2\Pi_{\mathring{d}5}^{\circ}] \quad .\ .\ .\ .\ .\ (22)$$

with

$$\left.\begin{aligned}
\Pi_{\mathring{d}1} &= 0\\
\Pi_{\mathring{d}2}^{\circ} &= \Phi_1\operatorname{cotg}\frac{\Theta}{2}\\
\Pi_{\mathring{d}3}^{\circ} &= 0\\
\Pi_{\mathring{d}4}^{\circ} &= -2\sin\varphi\operatorname{cotg}\frac{\Theta}{2}-2\int_{\varphi}^{\pi}\frac{\sin\Theta}{\cos\vartheta-\cos\Theta}d\vartheta\\
\Pi_{\mathring{d}5}^{\circ} &= 2\int_{\omega}^{\pi}L(\Theta,\vartheta)\sin\vartheta\, d\vartheta
\end{aligned}\right\}$$

and, after integration:

$$\Pi_{\mathring{d}4}^{\circ} = -2\sin\varphi\operatorname{cotg}\frac{\Theta}{2}+2L(\Theta,\varphi)$$

$$\Pi_{\mathring{d}5}^{\circ} = +2(\pi-\varphi)\sin\Theta+2(\cos\varphi-\cos\Theta)\,L(\varphi,\Theta).$$

Integration according to equations (9) and (10) yields the coefficients $k_{\mathring{d}}$, $m_{\mathring{d}}$, $r_{\mathring{d}}$, $n_{\mathring{d}}$, $p_{\mathring{d}}$, $q_{\mathring{d}}$, the last two dependent upon both φ and ψ; it is found that

$$\left.\begin{aligned}
\pi k_{\breve{d}}(\varphi) &= (1+T)\omega\Phi_1+\omega^2\Phi_3\\
\pi m_{\mathring{d}}^{\circ}(\varphi) &= \omega\Phi_5+\omega^2\frac{1}{4}\Phi_6\\
\pi^2 r_{\mathring{d}}^{\circ}(\varphi) &= (1+T)\omega\Phi_1\Phi_{31}+\omega\Phi_{35}+\omega^2\Phi_{17}\\
\pi^2 n_{\mathring{d}}^{\circ}(\varphi) &= (1+T)\omega\frac{1}{2}\Phi_1\Phi_8+\omega\Phi_{10}+\dot{\omega}^2\cdot\frac{1}{2}\Phi_{37}
\end{aligned}\right\}\quad(23)$$

$$\left.\begin{aligned}
\pi^2 p_{\mathring{d}}(\varphi,\psi) &= (1+T)\omega X_1+\omega X_3+\omega^2 X_{14}\\
\pi^2 q_{\mathring{i}}^{\circ}(\varphi,\psi) &= (1+T)\omega X_6+\omega X_8+\omega^2 X_{18}
\end{aligned}\right\}\quad .\ .\ .\ (24)$$

Functions $\Phi_i = \Phi_i(\varphi)$ are taken from the list in section VII, 5, and the functions $X_i = X_i(\varphi, \psi)$ from that in section VII, 6.

$\bar{d}$. Closed stage.

Its mean line is the closed oblique stage (fig. 3b). It can be combined from two plates with single bend:

$$z = z_c(C, \varphi - \delta_R;\ \Theta) + z_c(-C, \varphi;\ \Theta).$$

The height D of the stage for $\Theta = \varphi$ is:

$$D = C \cdot [\cos(\varphi - \delta_R) - \cos\varphi],$$

whence

$$z = \frac{z_c(D, \varphi - \delta_R;\ \Theta) - z_c(D, \varphi;\ \Theta)}{\cos(\varphi - \delta_R) - \cos\varphi}$$

Since the "gap," spanned here by the diagonal line, is very narrow, i.e., δ_R is very small, it suggests the replacement of the diagonal stage by the vertical stage created through $\delta_R \to 0$ (fig. 3b).

Then the pressure distribution is:

$$\Pi_{\bar{d}}(D, \varphi;\ \Theta, t) = \lim_{\delta_R \to 0} \frac{\Pi_c(D, \varphi - \delta_R;\ \Theta, t) - \Pi_c(D, \varphi;\ \Theta, t)}{\cos(\varphi - \delta_R) - \cos\varphi} = -\frac{1}{\sin\varphi}\frac{\partial \Pi_c(D, \varphi;\ \Theta, t)}{\partial\varphi}$$

Effecting this differentiation with the aid of the formulas for Π_c, yields $\Pi_{\bar{d}}$:

$$\Pi_{\bar{d}}(D, \varphi;\ \Theta, t) = \varrho v^2 e^{i\nu t} \cdot \frac{D}{\pi} \times [(1+T)\Pi_{\bar{d}1} + (1+T)\omega\Pi_{\bar{d}2} + \Pi_{\bar{d}3} + \omega\Pi_{\bar{d}4} + \omega^2\Pi_{\bar{d}5}], \tag{25}$$

whereby

$$\Pi_{\bar{d}i} = -\frac{1}{\sin\varphi}\frac{\partial \Pi_{ci}}{\partial\varphi}; \quad i = 1, \ldots, 5;$$

individually:

$$\Pi_{\bar{d}1} = \operatorname{tg}\frac{\varphi}{2}\operatorname{cotg}\frac{\Theta}{2}$$

$$\Pi_{\bar{d}2} = \Phi_1 \operatorname{cotg}\frac{\Theta}{2}$$

$$\Pi_{\bar{d}3} = 2\operatorname{cotg}\varphi \cdot \operatorname{cotg}\frac{\Theta}{2} - \frac{2}{\sin\varphi} \cdot \frac{\sin\Theta}{\cos\varphi - \cos\Theta}$$

$$\Pi_{\bar{d}4} = -2\sin\varphi \cdot \operatorname{cotg}\frac{\Theta}{2} + 4L(\varphi, \Theta)$$

$$\Pi_{\bar{d}5} = 2(\pi - \varphi)\sin\Theta + 2L(\varphi, \Theta)(\cos\varphi - \cos\Theta).$$

The forces and moments for wing and elevator integrated according to equations (9) and (10) are:

$$\left.\begin{aligned}
\pi k_{\bar{d}}(\varphi) &= (\Phi_{13} + \omega\Phi_1)(1+T) + \omega\Phi_{14} + \omega^2\Phi_3 \\
\pi m_{\bar{d}}(\varphi) &= \Phi_{15} + 2\omega\Phi_5 + \frac{1}{4}\omega^2\Phi_6 \\
\pi^2 r_{\bar{d}}(\varphi) &= (\Phi_{13} + \omega\Phi_1)\Phi_{31}(1+T) \\
&\qquad + \pi^2 r_{\bar{d}}^{*} + \omega\Phi_{16} + \omega^2\Phi_{17} \\
\pi^2 n_{\bar{d}}(\varphi) &= \frac{1}{2}\Phi_8(\Phi_{13} + \omega\Phi_1)(1+T) \\
&\qquad + \Phi_{18} + \omega\Phi_{19} + \frac{1}{2}\omega^2\Phi_{37}
\end{aligned}\right\} \tag{26}$$

Here the term $\pi^2 r_{\bar{d}}$ free from ω, expressed with $\pi^2 r_{\bar{d}}^{*}$, presents a certain obstacle. It had been assumed that the air does not pass through the narrow slot between fin and elevator. To insure this in theory, the gap was covered from fin trailing edge to elevator leading edge. Therefore the diagonal stage for the degree of freedom $\bar{d}$ was really obtained; this, however, was idealized to the vertical closed stage. The force and moment coefficients computed for the latter disclosed the summand in $r_{\bar{d}}$, $r_{\bar{d}}^{*}$, free from ω, as ∞, a sign that the idealization is carried too far. So, for computing this summand, the oblique stage was reverted to for which the term

$$\pi^2 r_{\bar{d}}^{*} = 2\ln\tau_{SR} + \Phi_{21},$$

was obtained; while it is noted that

$$\delta_R \sim \frac{2\tau_{SR}}{\sin\varphi}$$

by ommission of terms which become zero with disappearing gap width.

This difficulty arises in the stationary case ($\omega = 0$) also; hence it represents no peculiarity of the nonstationary calculation. The term $r_{\bar{d}}^{*}$ could equally well be taken from stationary measurements or from another than linearized stationary theory.

The factors $p_{\bar{d}}$ and $q_{\bar{d}}$, related to φ and ψ, are obtained on the basis of the vertical stage.

$$\left.\begin{aligned}
\pi^2 p_{\bar{d}}(\varphi, \psi) &= (1+T)X_{11} + (1+T)\omega X_1 \\
&\qquad + X_{12} + \omega X_{13} + \omega^2 X_{14} \\
\pi^2 q_{\bar{d}}(\varphi, \psi) &= (1+T)X_{15} + (1+T)\omega X_6 \\
&\qquad + X_{16} + \omega X_{17} + \omega^2 X_{18}
\end{aligned}\right\} \tag{22}$$

The functions $X_i = X_i(\varphi, \psi)$ are found in section VII, 6. The incomplete execution of $\delta_R \longrightarrow 0$ can be mathematically interpreted as follows:

The oblique stage (fig. 3b) serves as the basis of the plate deformation of d. In the calculation, however, of the pressure and of the aerodynamic coefficients, the terms that disappear by $\delta_R \longrightarrow 0$ are discounted.

Nevertheless, since all coefficients as far as the term $r_{\bar{d}}^*$ of $r_{\bar{d}}$ prove themselves to be independent of the slot width, the degree of freedom $\bar{d}$ is characterized in compilation VII by the vertical stage.

A proof for the calculation of quantities $k_{\bar{d}}$, $m_{\bar{d}}$, $r_{\bar{d}}$, $n_{\bar{d}}$, $p_{\bar{d}}$, and $q_{\bar{d}}$ is afforded when $\Pi_{\bar{d}}$ is obtained from Π_c by differentiation. Then the following formulas result:

$$k_{\bar{d}} = -\frac{1}{\sin\varphi}\frac{\partial k_c}{\partial\varphi}$$

$$m_{\bar{d}} = -\frac{1}{\sin\varphi}\frac{\partial m_c}{\partial\varphi}$$

$$\pi^2 r_{\bar{d}} = -\frac{1}{\sin\varphi}\frac{\partial \pi^2 r_c}{\partial\varphi} - [(1+T)\,\Pi_{c1} + (1+T)\,\omega\,\Pi_{c2} + \Pi_{c3} + \omega\,\Pi_{c4} + \omega^2\,\Pi_{c5}]_{\Theta=\varphi}$$

$$n_{\bar{d}} = -\frac{1}{\sin\varphi}\frac{\partial n_c}{\partial\varphi} - r_c$$

$$p_{\bar{d}} = -\frac{1}{\sin\varphi}\frac{\partial p_c}{\partial\varphi}$$

$$q_{\bar{d}} = -\frac{1}{\sin\varphi}\frac{\partial q_c}{\partial\varphi}$$

As an example, prove the fourth of the formulas. It is:

$$-\frac{1}{\sin\varphi}\frac{\partial}{\partial\varphi}\int_\varphi^\pi \Pi_c(D,\varphi;\ \Theta,t)\cdot(\cos\varphi - \cos\Theta)\sin\Theta\, d\Theta$$

$$= -\frac{1}{\sin\varphi}\cdot[-\Pi_c(D,\varphi;\Theta,t)\cdot(\cos\varphi-\cos\Theta)\sin\Theta]_{\Theta=\varphi}$$

$$+\int_\varphi^\pi \frac{-1}{\sin\varphi}\frac{\partial}{\partial\varphi}[\Pi_c(D,\varphi;\Theta,t)\cdot(\cos\varphi-\cos\Theta)]\sin\Theta\, d\Theta$$

$$= \int_\varphi^\pi \Pi_{\bar{d}}(D,\varphi;\ \Theta,t)\cdot(\cos\varphi-\cos\Theta)\sin\Theta\, d\Theta$$

$$+\int_\varphi^\pi \Pi_c(D,\varphi;\ \Theta,t)\sin\Theta\, d\Theta,$$

multiplied by $l^2 b$

$$-\frac{1}{\sin\varphi}\frac{\partial}{\partial\varphi}N_c(D,\varphi,t) = N_{\bar{d}}(D,\varphi,t) + l\cdot R_c(D,\varphi,t)$$

or divided by $\pi\rho v^2 l^2 b D e^{i\nu t}$

$$-\frac{1}{\sin\varphi}\frac{\partial n_c}{\partial\varphi} = n_{\bar{d}} + r_c.$$

e) Torsional Oscillation of Tab

f) Stage Oscillation of Tab

The two degrees of freedom, e and f, are easily obtained from c and d, for e and f refer in the same manner to the tab as c and d to the elevator. Hence, on passing from c to e and from d to f, C is replaced by E, D by F, τ_{SR} by τ_{SH}; elevator and tab exchange parts, whence φ, r, n are exchanged for ψ, p, q. Then the aerodynamic coefficients read:

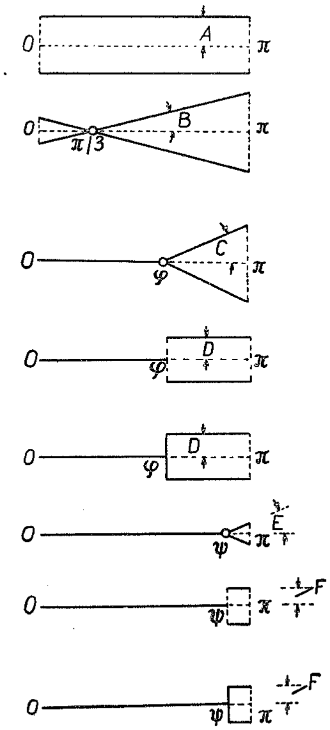
$$\left.\begin{aligned} k_e(\psi) &= k_c(\psi) & k_f(\psi) &= k_d(\psi)\\ m_e(\psi) &= m_c(\psi) & m_f(\psi) &= m_d(\psi)\\ r_e(\varphi,\psi) &= p_c(\psi,\varphi) & r_f(\varphi,\psi) &= p_d(\psi,\varphi)\\ n_e(\varphi,\psi) &= q_c(\psi,\varphi) & n_f(\varphi,\psi) &= q_d(\psi,\varphi)\\ p_e(\psi) &= r_c(\psi) & p_f(\psi) &= r_d(\psi)\\ q_e(\psi) &= n_c(\psi) & q_f(\psi) &= n_d(\psi)\end{aligned}\right\}\quad . \ . \ (23)$$

In the formulas containing the coefficients of d and f, the open and the closed stage, respectively, serve as a basis for elevator and tab.

VII. CORRELATION OF RESULTS

Figure 6 illustrates the six degrees of freedom, the closed stages of d and f being shown vertical for simplicity. An arbitrary degree of freedom is denoted with g, its amplitude with G. The concept "elevator torsional oscillation" is, be it noted, included. The actual torsional oscillation of the elevator with aerodynamic balance consists of elevator torsional and elevator stage oscillation (degrees of freedom c and d).

Section V contains the aerodynamic coefficients (13) serving for the prediction of the forces and moments (14). These coefficients are then tabulated in figure 4. Their relationship with the geometrical parameters φ and ψ is given in figure 5. These coefficients are subsequently compiled (sec. VI) and correlated in the following, the arrangement being effected on the basis of the relationship of φ and ψ.



a flapping oscillation of total wing (pure translation).

b torsional oscillation of wing about forward neutral pain $\left(\theta = \frac{\pi}{3},\ x = -\frac{1}{2}\right)$.

c torsional oscillation of elevator about its landing edge.

$\mathring{d}$ stage oscillation of elevator (pure translation) by open stage.

$\bar{d}$ stage oscillation of elevator (pure translation) by closed stage.

e torsional oscillation of tab about its landing edge.

$\mathring{f}$ stage oscillation of tab (pure translation) by open stage.

$\bar{f}$ stage oscillation of tab (pure translation) by closed stage.

Figure 6.-Correlation of the 6 degrees of freedom.

1. Coefficients independent of φ and ψ

$$k_a = (1+T)\,\omega + \omega^2$$

$$m_a = \frac{1}{2}\,\omega^2$$

$$k_b = (1+T)\,(1+\omega) + \omega + \frac{1}{2}\,\omega^2$$

$$m_b = \omega + \frac{3}{8}\,\omega^2.$$

2. Coefficients dependent upon φ only

$$\pi\,k_c\,(\varphi) = (1+T)\left(\Phi_1 + \frac{1}{2}\,\omega\,\Phi_2\right) + \omega\,\Phi_3 + \frac{1}{2}\,\omega^2\,\Phi_4$$

$$\pi\,m_c\,(\varphi) = \Phi_5 + \frac{1}{2}\,\omega\,\Phi_6 + \frac{1}{4}\,\omega^2\,\Phi_7$$

$$\pi\,k_{\mathring{d}}\,(\varphi) = (1+T)\,\omega\,\Phi_1 + \omega^2\,\Phi_3$$

$$\pi\,k_{\bar{d}}\,(\varphi) = (1+T)\,(\Phi_{13} + \omega\,\Phi_1) + \Phi_{14}\,\omega + \Phi_3\,\omega^2$$

$$\pi\,m_{\mathring{d}}\,(\varphi) = \omega\,\Phi_5 + \frac{1}{4}\,\omega^2\,\Phi_6$$

$$\pi\,m_{\bar{d}}\,(\varphi) = \Phi_{15} + 2\,\omega\,\Phi_5 + \frac{1}{4}\,\omega^2\,\Phi_6$$

$$\pi\,r_a\,(\varphi) = (1+T)\,\omega\,\Phi_{31} + \omega^2\,\Phi_3$$

$$\pi\,n_a\,(\varphi) = (1+T)\,\omega\,\frac{1}{2}\,\Phi_8 + \frac{1}{2}\,\omega^2\,\Phi_4$$

$$\pi\,r_b\,(\varphi) = (1+T)\,(1+\omega)\,\Phi_{31} + \omega\,\Phi_{32} + \frac{1}{4}\,\omega^2\,\Phi_6$$

$$\pi\,n_b\,(\varphi) = (1+T)\,(1+\omega)\,\frac{1}{2}\,\Phi_8 + \frac{1}{2}\,\omega\,\Phi_9 + \frac{1}{4}\,\omega^2\,\Phi_7$$

$$\pi^2\,r_c\,(\varphi) = (1+T)\left(\Phi_1 + \frac{1}{2}\,\omega\,\Phi_2\right)\Phi_{31} + \Phi_{35} + \omega\,\Phi_{36} + \frac{1}{2}\,\omega^2\,\Phi_{37}$$

$$\pi^2\,n_c\,(\varphi) = (1+T)\left(\Phi_1 + \frac{1}{2}\,\omega\,\Phi_2\right)\frac{1}{2}\,\Phi_8 + \Phi_{10} + \frac{1}{2}\,\omega\,\Phi_{11} + \frac{1}{4}\,\omega^2\,\Phi_{12}$$

$$\pi^2\,r_{\mathring{d}}\,(\varphi) = (1+T)\,\omega\,\Phi_1\,\Phi_{31} + \omega\,\Phi_{35} + \omega^2\,\Phi_{17}$$

$$\pi^2\,r_{\bar{d}}\,(\varphi) = (1+T)\,(\Phi_{13} + \omega\,\Phi_1)\,\Phi_{31} + (2\ln\tau_{SR} + \Phi_{21}) + \omega\,\Phi_{16} + \omega^2\,\Phi_{17}$$

$$\pi^2\,n_{\mathring{d}}\,(\varphi) = (1+T)\,\omega\,\frac{1}{2}\,\Phi_1\,\Phi_8 + \omega\,\Phi_{10} + \frac{1}{2}\,\omega^2\,\Phi_{37}$$

$$\pi^2\,n_{\bar{d}}\,(\varphi) = (1+T)\,(\Phi_{13} + \omega\,\Phi_1)\,\frac{1}{2}\,\Phi_8 + \Phi_{18} + \omega\,\Phi_{19} + \frac{1}{2}\,\omega^2\,\Phi_{37}.$$

The functions $\Phi_i = \Phi_i(\varphi)$ are read from VII, 5. The appearance of τ_{SR} (ratio of elevator gap width to wing chord) is explained in the formula for $r_{\bar{d}}$ in section VI.

3. Coefficients dependent upon ψ only

These are the same as for φ, except that ψ replaces φ and τ_{SH} substitutes for τ_{SR}.

$$p_a(\psi) = r_a(\psi) \qquad q_a(\psi) = n_a(\psi)$$
$$p_b(\psi) = r_b(\psi) \qquad q_b(\psi) = n_b(\psi)$$
$$k_e(\psi) = k_c(\psi) \qquad m_e(\psi) = m_c(\psi)$$
$$k_f(\psi) = k_d(\psi) \qquad m_f(\psi) = m_d(\psi)\ ^{2)}$$
$$p_e(\psi) = r_c(\psi) \qquad q_e(\psi) = n_c(\psi)$$
$$p_f(\psi) = r_d(\psi) \qquad q_f(\psi) = n_d(\psi)\ ^{2)}.$$

4. Coefficients dependent upon φ and ψ

$$\pi^2 p_c(\varphi,\psi) = (1+T)X_1 + (1+T)\omega X_2 + X_3 + \omega X_4 + \omega^2 X_5$$
$$\pi^2 q_c(\varphi,\psi) = (1+T)X_6 + (1+T)\omega X_7 + X_8 + \omega X_9 + \omega^2 X_{10}$$
$$\pi^2 p_{\mathring{d}}(\varphi,\psi) = (1+T)\omega X_1 + \omega X_3 + \omega^2 X_{14}$$
$$\pi^2 p_{\bar{d}}(\varphi,\psi) = (1+T)X_{11} + (1+T)\omega X_1 + X_{12} + \omega X_{13} + \omega^2 X_{14}$$
$$\pi^2 q_{\mathring{d}}(\varphi,\psi) = (1+T)\omega X_6 + \omega X_8 + \omega^2 X_{18}$$

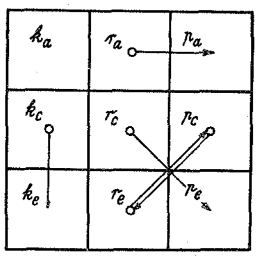


Figure 7.-Relation of coefficients to each other, arrow indicates replacement of φ by ψ.

$$\pi^2 q_{\bar{d}}(\varphi,\psi) = (1+T)X_{15} + (1+T)\omega X_6 + X_{16} + \omega X_{17} + \omega^2 X_{18}.$$

the functions $X_i = X_i(\varphi, \psi)$ to be taken from the list of the X functions.

The remaining four functions are obtained by exchanging φ and ψ:

$$r_e(\varphi,\psi) = p_c(\psi,\varphi) \qquad n_e(\varphi,\psi) = q_c(\psi,\varphi)$$
$$r_f(\varphi,\psi) = p_d(\psi,\varphi) \qquad n_f(\varphi,\psi) = q_d(\psi,\varphi)\ ^{2)}.$$

[2]Again it is to be borne in mind that in the formulas connecting d with f, the same type of stage is meant for elevator and tab, that is, both times the open or both times the closed stage. Naturally, this does not preclude the use of one type of stage for the wing with elevator and tab and the other type for the tab.

Lastly, the functions as they were evolved (fig. 7) are considered. The functions of field p_a were evolved from those of field r_a by substituting ψ for φ, field k_e from k_c and field p_e from field r_c. Functions of field p_c are obtained from those of field r_e by substituting ψ for φ.

The aerodynamic coefficients so obtained contain functions Φ and X, which have been computed for the practical zones of parameters:

5. Φ functions

$$\Phi_1(\varphi) = \pi - \varphi + \sin\varphi$$
$$\Phi_2(\varphi) = (\pi-\varphi)(1+2\cos\varphi) + \sin\varphi\,(2+\cos\varphi)$$
$$\Phi_3(\varphi) = \pi - \varphi + \sin\varphi\cos\varphi$$
$$\Phi_4(\varphi) = (\pi-\varphi)\cdot 2\cos\varphi + \sin\varphi\cdot\frac{2}{3}(2+\cos^2\varphi)$$
$$\Phi_5(\varphi) = \sin\varphi\cdot(1-\cos\varphi)$$
$$\Phi_6(\varphi) = 2(\pi-\varphi) + \sin\varphi\cdot\frac{2}{3}(2-\cos\varphi)(1+2\cos\varphi)$$
$$\Phi_7(\varphi) = (\pi-\varphi)\left(\frac{1}{2}+2\cos\varphi\right) + \sin\varphi\cdot\frac{1}{6}(8+5\cos\varphi+4\cos^2\varphi-2\cos^3\varphi)$$
$$\Phi_8(\varphi) = (\pi-\varphi)(-1+2\cos\varphi) + \sin\varphi\,(2-\cos\varphi)$$
$$\Phi_9(\varphi) = (\pi-\varphi)(1+2\cos\varphi) + \sin\varphi\cdot\frac{1}{3}(2+3\cos\varphi+4\cos^2\varphi)$$
$$\Phi_{10}(\varphi) = \Phi_{31}(\varphi)\cdot\Phi_5(\varphi)$$
$$\Phi_{11}(\varphi) = \Phi_2(\varphi)\cdot\Phi_3(\varphi)$$
$$\Phi_{12}(\varphi) = (\pi-\varphi)^2\left(\frac{1}{2}+4\cos^2\varphi\right) + (\pi-\varphi)\sin\varphi\cos\varphi\cdot(7+2\cos^2\varphi) + \sin^2\varphi\left(2+\frac{5}{2}\cos^2\varphi\right)$$
$$\Phi_{13}(\varphi) = \mathrm{tg}\,\frac{\varphi}{2}$$
$$\Phi_{14}(\varphi) = 2\sin\varphi$$
$$\Phi_{15}(\varphi) = \Phi_{13}(\varphi) - \Phi_{14}(\varphi)$$
$$\Phi_{16}(\varphi) = \Phi_1(\varphi)\cdot\Phi_{14}(\varphi) = 2\,\Phi_1(\varphi)\cdot\sin\varphi$$
$$\Phi_{17}(\varphi) = [\Phi_3(\varphi)]^2 + \sin^4\varphi$$
$$\Phi_{18}(\varphi) = -\Phi_{13}(\varphi)\cdot[(\pi-\varphi)(1+2\cos\varphi) - \sin\varphi\cdot\cos\varphi]$$
$$\Phi_{19}(\varphi) = \frac{1}{2}\,\Phi_3(\varphi)\cdot\Phi_{14}(\varphi) = \Phi_3(\varphi)\cdot\sin\varphi$$
$$\Phi_{20}(\varphi) = \sin\varphi\,(1+\cos\varphi)$$
$$\Phi_{21}(\varphi) = -2(\cos\varphi + \ln\sin^2\varphi)$$
$$\Phi_{31}(\varphi) = \pi - \varphi - \sin\varphi$$
$$\Phi_{32}(\varphi) = \pi - \varphi + \sin\varphi\,(1+2\cos\varphi)$$
$$\Phi_{35}(\varphi) = 2\sin^2\varphi$$
$$\Phi_{36}(\varphi) = \Phi_{32}(\varphi)\cdot\Phi_3(\varphi) + 2\sin^4\varphi$$
$$\Phi_{37}(\varphi) = \Phi_3(\varphi)\cdot[\Phi_2(\varphi) - \Phi_3(\varphi)].$$

Φ_1 to Φ_{12} was introduced by the first-named author (reference 1); Φ_{31} to Φ_{37} by Dietze (reference 3); Φ_{13} to Φ_{21} are new.

6. X functions

$$L(\varphi,\psi) = \ln\left|\frac{\sin\frac{1}{2}(\varphi+\psi)}{\sin\frac{1}{2}(\varphi-\psi)}\right| = \frac{1}{2}\ln\frac{1-\cos(\varphi+\psi)}{1-\cos(\varphi-\psi)}$$

$$3X_0(\varphi,\psi) = [(\pi-\varphi)\sin^3\psi-(\pi-\psi)\sin^3\varphi] + (\cos\varphi-\cos\psi)\cdot[2\sin\varphi\sin\psi-(\cos\varphi-\cos\psi)^2 L(\varphi,\psi)]$$

$$X_1(\varphi,\psi) = \Phi_1(\varphi)\cdot\Phi_{31}(\psi)$$

$$X_2(\varphi,\psi) = \frac{1}{2}\Phi_2(\varphi)\cdot\Phi_{31}(\psi)$$

$$X_3(\varphi,\psi) = 2\sin\varphi\cdot\sin\psi-2(\cos\varphi-\cos\psi)L(\varphi,\psi)$$

$$X_4(\varphi,\psi) = \Phi_3(\varphi)\cdot\Phi_{32}(\psi)+2\Phi_5(\varphi)\cdot\Phi_{20}(\psi) + (\cos\varphi-\cos\psi)X_3(\varphi,\psi)$$

$$X_5(\varphi,\psi) = \frac{1}{2}[\Phi_2(\varphi)-\Phi_3(\varphi)]\cdot\Phi_3(\psi)+X_0(\varphi,\psi)$$

$$X_6(\varphi,\psi) = \frac{1}{2}\Phi_1(\varphi)\cdot\Phi_8(\psi)$$

$$X_7(\varphi,\psi) = \frac{1}{4}\Phi_2(\varphi)\cdot\Phi_8(\psi)$$

$$X_8(\varphi,\psi) = \Phi_5(\varphi)\cdot\Phi_{31}(\psi)-\frac{1}{2}(\cos\varphi-\cos\psi)X_3(\varphi,\psi)$$

$$X_9(\varphi,\psi) = \frac{1}{2}\Phi_3(\varphi)\cdot\Phi_2(\psi)-2X_0(\varphi,\psi)$$

$$\begin{aligned}4X_{10}(\varphi,\psi) = {} & (\pi-\varphi)(\pi-\psi)\left(\frac{1}{2}+4\cos\varphi\cos\psi\right) \\ & +(\pi-\varphi)\sin\psi\left(3\cos\varphi+\frac{1}{2}\cos\psi+\cos\varphi\cos^2\psi\right) \\ & +(\pi-\psi)\sin\varphi\left(3\cos\psi+\frac{1}{2}\cos\varphi+\cos\psi\cos^2\varphi\right) \\ & +\sin\varphi\sin\psi\left[2+\frac{5}{2}\cos\varphi\cos\psi+(\cos\varphi-\cos\psi)^2\right] \\ & -(\cos\varphi-\cos\psi)X_0(\varphi,\psi)\end{aligned}$$

$$X_{11}(\varphi,\psi) = \Phi_{13}(\varphi)\cdot\Phi_{31}(\psi)$$

$$X_{12}(\varphi,\psi) = \Phi_{13}(\varphi)\cdot\Phi_{16}(\psi)-2L(\varphi,\psi)$$

$$X_{13}(\varphi,\psi) = \Phi_{16}(\varphi)\cdot\Phi_{31}(\psi)+2X_3(\varphi,\psi)$$

$$X_{14}(\varphi,\psi) = \Phi_3(\varphi)\cdot\Phi_3(\psi)+\Phi_5(\varphi)\cdot\Phi_{20}(\psi) + \frac{1}{2}(\cos\varphi-\cos\psi)X_3(\varphi,\psi)$$

$$X_{15}(\varphi,\psi) = \frac{1}{2}\Phi_{13}(\varphi)\cdot\Phi_8(\psi)$$

$$X_{16}(\varphi,\psi) = \Phi_{13}(\varphi)\cdot\Phi_3(\psi)-[\Phi_{14}(\varphi)\cdot\Phi_{31}(\psi)+X_3(\varphi,\psi)]$$

$$X_{17}(\varphi,\psi) = \frac{1}{2}\Phi_{14}(\varphi)\cdot\Phi_3(\psi) - [\Phi_{14}(\varphi)\cdot\Phi_{31}(\psi)+X_3(\varphi,\psi)]\cdot(\cos\varphi-\cos\psi)$$

$$X_{18}(\varphi,\psi) = X_5(\psi,\varphi).$$

7. Notes on the numerical calculation of the Φ and X functions

On the conventional types of aircraft the values of τ_R and τ_H in the following intervals are appropriate:

$$0{,}6 \geq \tau_R \geq 0{,}1; \qquad 0{,}1 \geq \tau_H \geq 0{,}0.$$

hence the values

$$\tau_R = 0{,}60 \quad 0{,}55 \ldots\ldots 0{,}10$$
$$\tau_H = 0{,}10 \quad 0{,}09 \ldots\ldots 0{,}00.$$

are used as a basis for tabulating Φ and X. To it correspond φ and ψ with

$-\cos\varphi = 1-2\tau_R = -0.2 \quad -0.1 \quad 0 \quad 0.1 \ldots 0.8$

$-\cos\psi = 1-2\tau_H = 0.80 \quad 0.82 \ldots \ldots \ldots 1.00$

In the Φ functions 0.9 and 1.0 are added to the values of $-\cos\varphi$ and the range of $-\cos\psi$ then considered, as it were, as refined division of $-\cos\varphi$. When the tables are used, these values appear as values of $-\cos\psi$.

The X functions include also the two auxiliary functions $L(\varphi,\psi)$ and $X_0(\varphi,\psi)$ since they play a part in many X functions. Not tabulated are the functions X_i of the form

$$X_i(\varphi,\psi) = \Phi_n(\varphi)\ \Phi_m(\psi)$$

All the others are tabulated, including $X_i(\varphi,\psi)$ and $X_i(\psi,\varphi)$; the exchange of the arguments is superfluous for L, X_0, X_{10} X_{14} since these are partly symmetrical in the variables:

$$L(\varphi,\psi) = L(\psi,\varphi)$$
$$X_0(\varphi,\psi) = -X_0(\psi,\varphi)$$
$$X_{10}(\varphi,\psi) = X_{10}(\psi,\varphi)$$
$$X_{14}(\varphi,\psi) = X_{14}(\psi,\varphi)$$

The values of the subsequent 25 tables were computed to seven digits.

APPENDIX

The tabulation of the functions g, $\bar{g}$ by Dietze (reference 2) ties in with ours in the following manner:

$$g_1 = \frac{1}{2\pi}\,\Phi_4 \qquad g_{10} = \frac{1}{\pi^2}\,\Phi_{35}$$

$$g_2 = \frac{1}{2\pi}\,\Phi_2 \qquad g_{11} = \frac{1}{4\pi}\,(\Phi_4 + \Phi_7)$$

$$g_3 = \frac{1}{\pi}\,\Phi_3 \qquad g_{12} = \frac{1}{\pi}\,(\Phi_3 + \Phi_6)$$

$$g_4 = \frac{1}{\pi}\,\Phi_1 \qquad g_{13} = \frac{2}{\pi}\,\Phi_5$$

$$g_5 = \frac{1}{\pi}\,\Phi_{31} \qquad g_{14} = \frac{1}{2\pi}\,\Phi_8$$

$$g_6 = \frac{1}{4\pi}\left(\Phi_3 + \frac{1}{2}\,\Phi_6\right) \qquad g_{15} = \frac{1}{2\pi}\,\Phi_9$$

$$g_7 = \frac{1}{\pi}\,\Phi_{32} \qquad g_{16} = \frac{1}{4\pi^2}\,\Phi_{12}$$

$$g_8 = \frac{1}{2\pi^2}\,\Phi_{37} \qquad g_{17} = \frac{1}{2\pi^2}\,\Phi_{11}$$

$$g_9 = \frac{1}{\pi^2}\,\Phi_{36} \qquad g_{18} = \frac{1}{\pi^2}\,\Phi_{19}$$

Here the g_i's are to be taken for $\frac{t_\mu}{t_{Fl}}$ which correspond to our τ_R, the Φ_i's for the argument φ. In addition

$$\bar{g}_8 = \frac{1}{\pi^2} X_5 \qquad \bar{g}_{16} = \frac{1}{\pi^2} X_{10}$$

$$\bar{g}_9 = \frac{1}{\pi^2} X_4 \qquad \bar{g}_{17} = \frac{1}{\pi^2} X_9$$

$$\bar{g}_{10} = \frac{1}{\pi^2} X_3 \qquad \bar{g}_{18} = \frac{1}{\pi^2} X_8$$

where $\overline{g_i}$ is to be taken for $\frac{t_\mu}{t_{Fl}}$, $\frac{t_\nu}{t_{Fl}}$, that is, for τ_R, τ_H, but X_i for ψ, φ rather than for φ, ψ.

Dietze divides the deformation of the plate into four motion parts 0, 1, 2, and 3. Denoting these deformations with z_0, z_1, z_2, and z_3, with amplitudes $\underline{B}_0$, $\underline{B}_1$, $\underline{B}_2$, and $\underline{B}_3$ affords

$$z_0 = 2\, z_a\, (\underline{B}_0,\ \theta) = 2\, \underline{B}_0\, e^{i\nu t}$$

$$z_2 = z_c\, (\underline{B}_2,\ \theta)$$

$$z_3 = z_e\, (\underline{B}_3,\ \theta)$$

z_1 is wing rotation about the wing leading edge with amplitude $\underline{B}_1$; hence

$$z_1 = \frac{1}{2}\, \underline{B}_1\, e^{i\nu t} + z_b\, (\underline{B}_1,\ \theta) = -\, z_a\, (\underline{B}_1,\ \theta) + z_b(\underline{B}_1,\ \theta)$$

For a given form change of plate it must be $z_0 + z_1 + z_2 + z_3 = z_a + z_b + z_c + z_e$, whence the amplitudes follow at

$$\underline{B}_0 = \frac{1}{2} A - \frac{1}{4} B$$

$$\underline{B}_1 = B$$

$$\underline{B}_2 = C$$

Between the forces and moments of both reports the following relations exist:

$$P\ (x_v,\ x_h;\ t) = -K$$

$$P\ (x_2,\ x_h;\ t) = -R$$

$$P\ (x_3,\ x_h;\ t) = -P$$

$$M\ (x_v,\ x_h;\ x_v, t) = -\left(M_o + \frac{1}{2}K\right)$$

$$N\ (x_2,\ x_h;\ x_2, t) = -N$$

$$M\ (x_3,\ x_h;\ x_3, t) = -Q$$

where the moment M_o is referred to the forward neutral point, but $M(x_v,\ x_h;\ x_v,\ t)$ to the wing leading edge.

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Table 1. Functions $\Phi_1(\varphi)$ to $\Phi_8(\varphi)$.

τ_R	$-\cos\varphi$	Φ_1	Φ_2	Φ_3	Φ_4	Φ_5	Φ_6	Φ_7	Φ_8
0,60	− 0,2	2,75195	4,63657	1,96811	2,04138	0,78384	5,19037	3,08815	0,70034
0,55	− 0,1	2,66595	4,09463	1,77046	1,66748	0,89549	4,85431	2,58554	0,55371
0,50	0,0	2,57080	3,57080	1,57080	1,33333	1,00000	4,47493	2,11873	0,42920
0,45	0,1	2,46562	3,06698	1,37113	1,03916	1,09449	4,05564	1,69189	0,32472
0,40	0,2	2,34923	2,58530	1,17348	0,78475	1,17576	3,60110	1,30878	0,23834
0,35	0,3	2,22004	2,12814	0,97992	0,56949	1,24012	3,11729	0,97265	0,16829
0,30	0,4	2,07579	1,69828	0,79267	0,39236	1,28312	2,61184	0,68605	0,11293
0,25	0,5	1,91322	1,29904	0,61418	0,25184	1,29904	2,09440	0,45068	0,07067
0,20	0,6	1,72730	0,93454	0,44730	0,14591	1,28000	1,57726	0,26716	0,03995
0,15	0,7	1,50954	0,61023	0,29550	0,07192	1,21404	1,07661	0,13469	0,01923
0,10	0,8	1,24350	0,33390	0,16350	0,02640	1,08000	0,61500	0,05055	0,00690
0,05	0,9	0,88692	0,11866	0,05873	0,00472	0,82819	0,22788	0,00924	0,00121
0,00	1,0	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000
0,10	0,80	1,24350	0,33390	0,16350	0,02640	1,08000	0,61500	0,05055	0,00690
0,09	0,82	1,18175	0,28538	0,14005	0,02033	1,04170	0,53010	0,03910	0,00529
0,08	0,84	1,11610	0,23941	0,11774	0,01518	0,99836	0,44846	0,02932	0,00393
0,07	0,86	1,04582	0,19616	0,09667	0,01089	0,94915	0,37052	0,02114	0,00281
0,06	0,88	0,96991	0,15582	0,07696	0,00743	0,89295	0,29679	0,01447	0,00191
0,05	0,90	0,88692	0,11866	0,05873	0,00472	0,82819	0,22788	0,00924	0,00121
0,04	0,92	0,79463	0,08499	0,04215	0,00271	0,75248	0,16457	0,00532	0,00069
0,03	0,94	0,68934	0,05526	0,02746	0,00132	0,66188	0,10787	0,00261	0,00033
0,02	0,96	0,56379	0,03011	0,01499	0,00048	0,54880	0,05926	0,00095	0,00012
0,01	0,98	0,39933	0,01066	0,00532	0,00009	0,39402	0,02114	0,00017	0,00002
0,00	1,00	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000

Table 2. Functions $\Phi_9(\varphi)$ to $\Phi_{17}(\varphi)$.

τ_R	$-\cos\varphi$	Φ_9	Φ_{10}	Φ_{11}	Φ_{12}	Φ_{13}	Φ_{14}	Φ_{15}	Φ_{16}	Φ_{17}
0,60	− 0,2	3,38243	0,62108	9,12529	6,54742	0,81650	1,95959	−1,14310	5,39270	4,79507
0,55	− 0,1	2,78125	0,60533	7,24939	4,67963	0,90453	1,98997	−1,08544	5,30518	4,11464
0,50	0,0	2,23746	0,57080	5,60899	3,23370	1,00000	2,00000	−1,00000	5,14159	3,46740
0,45	0,1	1,75360	0,52058	4,20523	2,14543	1,10554	1,98997	−0,88443	4,90651	2,86010
0,40	0,2	1,33116	0,45812	3,03379	1,35379	1,22474	1,95959	−0,73485	4,60354	2,29865
0,35	0,3	0,97069	0,38712	2,08541	0,80178	1,36277	1,90788	−0,54511	4,23557	1,78835
0,30	0,4	0,67178	0,31150	1,34618	0,43709	1,52753	1,83303	−0,30551	3,80499	1,33393
0,25	0,5	0,43301	0,23535	0,79785	0,21281	1,73205	1,73205	0,00000	3,31380	0,93972
0,20	0,6	0,25187	0,16294	0,41802	0,08797	2,00000	1,60000	0,40000	2,76367	0,60967
0,15	0,7	0,12461	0,09865	0,18032	0,02809	2,38048	1,42829	0,95219	2,15606	0,34742
0,10	0,8	0,04590	0,04698	0,05459	0,00560	3,00000	1,20000	1,80000	1,49220	0,15633
0,05	0,9	0,00823	0,01254	0,00697	0,00035	4,35890	0,87178	3,48712	0,77320	0,03955
0,00	1,0	0,00000	0,00000	0,00000	0,00000	∞	0,00000	∞	0,00000	0,00000
0,10	0,80	0,04590	0,04698	0,05459	0,00560	3,00000	1,20000	1,80000	1,49220	0,15633
0,09	0,82	0,03537	0,03857	0,03997	0,00368	3,17980	1,14473	2,03507	1,35278	0,12694
0,08	0,84	0,02643	0,03088	0,02819	0,00230	3,39117	1,08517	2,30599	1,21116	0,10053
0,07	0,86	0,01898	0,02395	0,01896	0,00135	3,64496	1,02059	2,62437	1,06735	0,07715
0,06	0,88	0,01295	0,01782	0,01199	0,00073	3,95811	0,94995	3,00817	0,92136	0,05682
0,05	0,90	0,00823	0,01254	0,00697	0,00035	4,35890	0,87178	3,48712	0,77320	0,03955
0,04	0,92	0,00473	0,00812	0,00358	0,00014	4,89898	0,78384	4,11514	0,62286	0,02537
0,03	0,94	0,00231	0,00463	0,00152	0,00005	5,68624	0,68235	5,00389	0,47037	0,01430
0,02	0,96	0,00084	0,00208	0,00045	0,00001	7,00000	0,56000	6,44000	0,31573	0,00637
0,01	0,98	0,00015	0,00053	0,00006	0,00000	9,94987	0,39800	9,55188	0,15893	0,00160
0,00	1,00	0,00000	0,00000	0,00000	0,00000	∞	0,00000	∞	0,00000	0,00000

Table 3. Functions $\Phi_{18}(\varphi)$ to $\Phi_{37}(\varphi)$.

τ_R	$-\cos\varphi$	Φ_{18}	Φ_{19}	Φ_{20}	Φ_{21}	Φ_{31}	Φ_{32}	Φ_{35}	Φ_{36}	Φ_{37}
0,60	−0,2	1,86574	1,92835	1,17576	−0,31836	0,79236	3,14367	1,92000	8,03069	5,25182
0,55	−0,1	1,72373	1,76159	1,09449	−0,17990	0,67598	2,86495	1,98000	7,03248	4,11485
0,50	0,0	1,57080	1,57080	1,00000	0,00000	0,57080	2,57080	2,00000	6,03820	3,14159
0,45	0,1	1,41067	1,36426	0,89549	0,22010	0,47564	2,26662	1,98000	5,06803	2,32523
0,40	0,2	1,24633	1,14977	0,78384	0,48164	0,38964	1,95732	1,92000	4,14007	1,65674
0,35	0,3	1,08016	0,93479	0,66776	0,78862	0,31216	1,64768	1,82000	3,27080	1,12516
0,30	0,4	0,91417	0,72650	0,54991	1,14871	0,24276	1,34258	1,68000	2,47543	0,71785
0,25	0,5	0,75000	0,53190	0,43301	1,57536	0,18117	1,04720	1,50000	1,76817	0,42063
0,20	0,6	0,58908	0,35784	0,32000	2,09257	0,12730	0,76730	1,28000	1,16241	0,21794
0,15	0,7	0,43263	0,21103	0,21424	2,74669	0,08126	0,50974	1,02000	0,67083	0,09300
0,10	0,8	0,28170	0,09810	0,12000	3,64330	0,04350	0,28350	0,72000	0,30555	0,02786
0,05	0,9	0,13722	0,02560	0,04359	5,12146	0,01514	0,10231	0,38000	0,07821	0,00352
0,00	1,0	0,00000	0,00000	0,00000	∞	0,00000	0,00000	0,00000	0,00000	0,00000
0,10	0,80	0,28170	0,09810	0,12000	3,64330	0,04350	0,28350	0,72000	0,30555	0,02786
0,09	0,82	0,25226	0,08016	0,10303	3,87192	0,03702	0,24307	0,65520	0,24869	0,02035
0,08	0,84	0,22308	0,06388	0,08681	4,12563	0,03093	0,20455	0,58880	0,19743	0,01433
0,07	0,86	0,19418	0,04933	0,07144	4,41107	0,02523	0,16812	0,52080	0,15187	0,00962
0,06	0,88	0,16556	0,03655	0,05700	4,73798	0,01996	0,13395	0,45120	0,11210	0,00607
0,05	0,90	0,13722	0,02560	0,04359	5,12146	0,01514	0,10231	0,38000	0,07821	0,00352
0,04	0,92	0,10917	0,01652	0,03135	5,58681	0,01080	0,07350	0,30720	0,05028	0,00181
0,03	0,94	0,08142	0,00937	0,02047	6,18145	0,00699	0,04793	0,23280	0,02841	0,00076
0,02	0,96	0,05397	0,00420	0,01120	7,01186	0,00379	0,02619	0,15680	0,01269	0,00023
0,01	0,98	0,02683	0,00106	0,00398	8,41785	0,00134	0,00930	0,07920	0,00319	0,00003
0,00	1,00	0,00000	0,00000	0,00000	∞	0,00000	0,00000	0,00000	0,00000	0,00000

Table 4. Function $L(\varphi,\psi) = L(\psi,\varphi)$.

$-\cos\psi$	$-\cos\varphi$	−0,2	−0,1	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8
	τ_H \ τ_R	0,60	0,55	0,50	0,45	0,40	0,35	0,30	0,25	0,20	0,15	0,10
0,80	0,10	0,55840	0,62236	0,69315	0,77340	0,86701	0,98010	1,12323	1,31696	1,60944	2,18158	∞
0,82	0,09	0,52531	0,58506	0,65103	0,72680	0,81222	0,91629	1,04697	1,22164	1,47942	1,93964	3,53721
0,84	0,08	0,49119	0,54669	0,60781	0,67671	0,75643	0,85173	0,97051	1,12750	1,35462	1,74232	2,79354
0,86	0,07	0,45574	0,50690	0,56313	0,62632	0,69818	0,78585	0,89315	1,03351	1,23305	1,56133	2,33243
0,88	0,06	0,41857	0,46527	0,51647	0,57387	0,63982	0,71792	0,81400	0,93850	1,11277	1,39073	1,98270
0,90	0,05	0,37911	0,42114	0,46715	0,51857	0,57748	0,64684	0,73189	0,84102	0,99166	1,22567	1,68924
0,92	0,04	0,33647	0,37356	0,41406	0,45924	0,51083	0,57140	0,64509	0,73900	0,86701	1,06139	1,42542
0,94	0,03	0,28918	0,32087	0,35542	0,39386	0,43763	0,48863	0,55078	0,62917	0,73482	0,89208	1,17360
0,96	0,02	0,23435	0,25989	0,28768	0,31854	0,35356	0,39440	0,44357	0,50536	0,58779	0,70834	0,91629
0,98	0,01	0,16449	0,18232	0,20169	0,22314	0,24744	0,27566	0,30949	0,35174	0,40756	0,48795	0,62236
1,00	0,00	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000

Table 5. Function $X_0(\varphi,\psi) = -X_0(\psi,\varphi)$.

$-\cos\psi$	$-\cos\varphi$	−0,2	−0,1	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8
	τ_H \ τ_R	0,60	0,55	0,50	0,45	0,40	0,35	0,30	0,25	0,20	0,15	0,10
0,80	0,10	0,13162	0,11598	0,10030	0,08476	0,06957	0,05491	0,04101	0,02815	0,01665	0,00699	0,00000
0,82	0,09	0,11522	0,10178	0,08829	0,07491	0,06180	0,04914	0,03710	0,02592	0,01586	0,00731	0,00091
0,84	0,08	0,09896	0,08762	0,07623	0,06492	0,05383	0,04309	0,03286	0,02333	0,01470	0,00730	0,00159
0,86	0,07	0,08298	0,07363	0,06424	0,05491	0,04574	0,03685	0,02837	0,02043	0,01321	0,00696	0,00202
0,88	0,06	0,06743	0,05997	0,05246	0,04499	0,03765	0,03051	0,02369	0,01729	0,01144	0,00632	0,00221
0,90	0,05	0,05251	0,04679	0,04104	0,03531	0,02967	0,02419	0,01893	0,01398	0,00944	0,00544	0,00216
0,92	0,04	0,03844	0,03433	0,03018	0,02605	0,02198	0,01801	0,01420	0,01061	0,00729	0,00435	0,00191
0,94	0,03	0,02554	0,02285	0,02014	0,01743	0,01477	0,01216	0,00966	0,00729	0,00509	0,00313	0,00148
0,96	0,02	0,01421	0,01274	0,01126	0,00977	0,00831	0,00687	0,00549	0,00419	0,00297	0,00188	0,00094
0,98	0,01	0,00514	0,00461	0,00408	0,00356	0,00303	0,00252	0,00203	0,00156	0,00112	0,00073	0,00038
1,00	0,00	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000

Table 6. Function $X_3(\varphi,\psi)$.

$-\cos\psi$	$-\cos\varphi$	−0,2	−0,1	0,0	0,1	0,2	0.3	0,4	0,5	0,6	0,7	0,8
	τ_H \ τ_R	0,60	0,55	0,50	0,45	0,40	0,35	0,30	0,25	0,20	0,15	0,10
0,80	0,10	0,05895	0,07373	0,09096	0,11122	0,13534	0,16463	0,20123	0,24906	0,31622	0,42466	0,72000
0,82	0,09	0,04997	0,06247	0,07704	0,09413	0,11445	0,13906	0,16971	0,20951	0,26484	0,35199	0,54535
0,84	0,08	0,04158	0,05196	0,06405	0,07821	0,09501	0,11532	0,14053	0,17309	0,21792	0,28712	0,42762
0,86	0,07	0,03379	0,04222	0,05201	0,06347	0,07705	0,09343	0,11369	0,13973	0,17529	0,22922	0,33246
0,88	0,06	0,02663	0,03326	0,04095	0,04995	0,06060	0,07340	0,08920	0,10942	0,13681	0,17774	0,25274
0,90	0,05	0,02012	0,02512	0,03092	0,03769	0,04569	0,05530	0,06711	0,08217	0,10243	0,13231	0,18522
0,92	0,04	0,01430	0,01785	0,02196	0,02675	0,03241	0,03919	0,04751	0,05806	0,07218	0,09276	0,12820
0,94	0,03	0,00923	0,01151	0,01416	0,01724	0,02087	0,02522	0,03054	0,03726	0,04620	0,05910	0,08080
0,96	0,02	0,00499	0,00622	0,00765	0,00931	0,01127	0,01360	0,01645	0,02004	0,02479	0,03158	0,04279
0,98	0,01	0,00175	0,00219	0,00269	0,00327	0,00395	0,00477	0,00576	0,00701	0,00865	0,01097	0,01475
1,00	0,00	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000

Table 7. Function $X_3(\psi, \varphi)$.

$-\cos\psi$	$-\cos\varphi$	—0,2	—0,1	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8
	τ_H \ τ_R	0,60	0,55	0,50	0,45	0,40	0,35	0,30	0,25	0,20	0,15	0,10
0,80	0,10	2,29256	2,31424	2,30904	2,27675	2,21617	2,12483	1,99840	1,82941	1,60378	1,28929	0,72000
0,82	0,09	2,19323	2,21550	2,21242	2,18385	2,12875	2,04494	1,92861	1,77321	1,56673	1,28301	0,82832
0,84	0,08	2,08492	2,10750	2,10630	2,08126	2,03148	1,95506	1,84863	1,70649	1,51836	1,26282	0,87459
0,86	0,07	1,96614	1,98873	1,98917	1,96747	1,92288	1,85373	1,75708	1,62798	1,45766	1,22847	0,89224
0,88	0,06	1,83488	1,85711	1,85894	1,84042	1,80091	1,73898	1,65208	1,53594	1,38311	1,17908	0,88720
0,90	0,05	1,68821	1,70970	1,71264	1,69713	1,66264	1,60795	1,53089	1,42780	1,29242	1,11284	0,86092
0,92	0,04	1,52170	1,54197	1,54572	1,53306	1,50359	1,45627	1,38929	1,29958	1,18196	1,02678	0,81240
0,94	0,03	1,32790	1,34635	1,35054	1,34062	1,31625	1,27662	1,22023	1,14460	1,04556	0,91549	0,73802
0,96	0,02	1,09238	1,10816	1,11235	1,10507	1,08610	1,05481	1,01004	0,94990	0,87121	0,76826	0,62921
0,98	0,01	0,77815	0,78981	0,79330	0,78873	0,77596	0,75456	0,72378	0,68234	0,62815	0,55748	0,46285
1,00	0,00	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000

Table 8. Function $X_4(\varphi, \psi)$.

$-\cos\psi$	$-\cos\varphi$	—0,2	—0,1	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8
	τ_H \ τ_R	0,60	0,55	0,50	0,45	0,40	0,35	0,30	0,25	0,20	0,15	0,10
0,80	0,10	0,80503	0,78320	0,75809	0,72925	0,69607	0,65775	0,61317	0,56061	0,49725	0,41761	0,30555
0,82	0,09	0,69088	0,67234	0,65104	0,62658	0,59846	0,56603	0,52834	0,48400	0,43073	0,36422	0,27318
0,84	0,08	0,58193	0,56648	0,54874	0,52838	0,50499	0,47804	0,44676	0,41003	0,36604	0,31143	0,23807
0,86	0,07	0,47869	0,46612	0,45169	0,43513	0,41613	0,39425	0,36889	0,33917	0,30366	0,25982	0,20175
0,88	0,06	0,38175	0,37183	0,36045	0,34740	0,33243	0,31521	0,29527	0,27193	0,24413	0,20997	0,16523
0,90	0,05	0,29184	0,28433	0,27572	0,26586	0,25455	0,24155	0,22652	0,20896	0,18808	0,16253	0,12940
0,92	0,04	0,20984	0,20449	0,19837	0,19135	0,18332	0,17409	0,16343	0,15099	0,13624	0,11826	0,09513
0,94	0,03	0,13695	0,13350	0,12954	0,12501	0,11983	0,11388	0,10702	0,09902	0,08955	0,07805	0,06337
0,96	0,02	0,07490	0,07303	0,07089	0,06844	0,06564	0,06242	0,05872	0,05441	0,04931	0,04315	0,03532
0,98	0,01	0,02661	0,02595	0,02520	0,02434	0,02335	0,02222	0,02092	0,01941	0,01763	0,01548	0,01277
1,00	0,00	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000

Table 9. Function $X_4(\psi, \varphi)$.

$-\cos\psi$	$-\cos\varphi$	—0,2	—0,1	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8
	τ_H \ τ_R	0,60	0,55	0,50	0,45	0,40	0,35	0,30	0,25	0,20	0,15	0,10
0,80	0,10	0,76110	0,74676	0,73310	0,71112	0,68341	0,64934	0,60796	0,55770	0,49590	0,41718	0,30555
0,82	0,09	0,65277	0,64070	0,62925	0,61073	0,58734	0,55859	0,52369	0,48137	0,42947	0,36378	0,27315
0,84	0,08	0,54950	0,53953	0,53011	0,51478	0,49541	0,47159	0,44269	0,40770	0,36489	0,31100	0,23800
0,86	0,07	0,45175	0,44370	0,43614	0,42374	0,40807	0,38880	0,36542	0,33715	0,30264	0,25942	0,20167
0,88	0,06	0,36006	0,35377	0,34787	0,33816	0,32586	0,31074	0,29241	0,27025	0,24327	0,20961	0,16515
0,90	0,05	0,27510	0,27038	0,26598	0,25868	0,24943	0,23805	0,22426	0,20761	0,18738	0,16223	0,12932
0,92	0,04	0,19769	0,19436	0,19127	0,18611	0,17957	0,17151	0,16176	0,14999	0,13570	0,11802	0,09506
0,94	0,03	0,12895	0,12682	0,12485	0,12154	0,11733	0,11216	0,10589	0,09834	0,08918	0,07789	0,06331
0,96	0,02	0,07049	0,06934	0,06829	0,06651	0,06425	0,06146	0,05809	0,05402	0,04910	0,04305	0,03529
0,98	0,01	0,02503	0,02463	0,02426	0,02364	0,02285	0,02187	0,02069	0,01927	0,01755	0,01545	0,01276
1,00	0,00	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000

Table 10. Function $X_5(\varphi, \psi)$.

$-\cos\psi$	$-\cos\varphi$	—0,2	—0,1	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8
	τ_H \ τ_R	0,60	0,55	0,50	0,45	0,40	0,35	0,30	0,25	0,20	0,15	0,10
0,80	0,10	0,34977	0,30598	0,26380	0,22340	0,18498	0,14877	0,11504	0,08413	0,05648	0,03272	0,01393
0,82	0,09	0,30208	0,26453	0,22834	0,19366	0,16066	0,12954	0,10051	0,07387	0,04998	0,02935	0,01285
0,84	0,08	0,25606	0,22445	0,19397	0,16476	0,13694	0,11069	0,08618	0,06364	0,04339	0,02582	0,01162
0,86	0,07	0,21196	0,18598	0,16092	0,13688	0,11398	0,09235	0,07214	0,05353	0,03677	0,02217	0,01026
0,88	0,06	0,17011	0,14940	0,12942	0,11024	0,09197	0,07470	0,05854	0,04364	0,03019	0,01843	0,00877
0,90	0,05	0,13086	0,11504	0,09977	0,08511	0,07113	0,05790	0,04552	0,03409	0,02375	0,01468	0,00717
0,92	0,04	0,09468	0,08331	0,07233	0,06179	0,05173	0,04221	0,03329	0,02504	0,01756	0,01099	0,00550
0,94	0,03	0,06218	0,05476	0,04760	0,04072	0,03415	0,02793	0,02209	0,01669	0,01178	0,00745	0,00382
0,96	0,02	0,03422	0,03017	0,02625	0,02249	0,01889	0,01548	0,01228	0,00932	0,00662	0,00424	0,00222
0,98	0,01	0,01223	0,01079	0,00940	0,00806	0,00679	0,00557	0,00444	0,00338	0,00242	0,00156	0,00084
1,00	0,00	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000

Table 11. Function $X_5(\psi, \varphi)$.

$-\cos\psi$	$-\cos\varphi$	−0,2	−0,1	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8
	$\tau_H \diagdown \tau_R$	0,60	0,55	0,50	0,45	0,40	0,35	0,30	0,25	0,20	0,15	0,10
0,80	0,10	0,03606	0,03486	0,03353	0,03205	0,03041	0,02858	0,02652	0,02418	0,02146	0,01819	0,01393
0,82	0,09	0,02779	0,02688	0,02586	0,02473	0,02347	0,02207	0,02050	0,01871	0,01664	0,01416	0,01097
0,84	0,08	0,02077	0,02009	0,01933	0,01849	0,01756	0,01652	0,01536	0,01404	0,01251	0,01068	0,00835
0,86	0,07	0,01492	0,01443	0,01389	0,01329	0,01263	0,01189	0,01106	0,01012	0,00904	0,00774	0,00611
0,88	0,06	0,01018	0,00985	0,00948	0,00908	0,00863	0,00813	0,00757	0,00693	0,00620	0,00533	0,00424
0,90	0,05	0,00647	0,00626	0,00603	0,00578	0,00549	0,00518	0,00482	0,00442	0,00396	0,00342	0,00274
0,92	0,04	0,00371	0,00360	0,00346	0,00332	0,00316	0,00298	0,00278	0,00255	0,00229	0,00198	0,00159
0,94	0,03	0,00181	0,00176	0,00169	0,00162	0,00154	0,00146	0,00136	0,00125	0,00112	0,00097	0,00079
0,96	0,02	0,00066	0,00064	0,00062	0,00059	0,00056	0,00053	0,00050	0,00046	0,00041	0,00036	0,00029
0,98	0,01	0,00012	0,00011	0,00011	0,00010	0,00010	0,00009	0,00009	0,00008	0,00007	0,00006	0,00005
1,00	0,00	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000

Table 12. Function $X_8(\varphi, \psi)$.

$-\cos\psi$	$-\cos\varphi$	−0,2	−0,1	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8
	$\tau_H \diagdown \tau_R$	0,60	0,55	0,50	0,45	0,40	0,35	0,30	0,25	0,20	0,15	0,10
0,80	0,10	0,00462	0,00578	0,00712	0,00868	0,01055	0,01279	0,01557	0,01915	0,02406	0,03158	0,04698
0,82	0,09	0,00353	0,00441	0,00544	0,00663	0,00805	0,00976	0,01187	0,01457	0,01826	0,02383	0,03453
0,84	0,08	0,00262	0,00327	0,00403	0,00491	0,00596	0,00722	0,00877	0,01075	0,01344	0,01745	0,02485
0,86	0,07	0,00187	0,00233	0,00287	0,00350	0,00424	0,00513	0,00623	0,00763	0,00951	0,01230	0,01728
0,88	0,06	0,00126	0,00158	0,00194	0,00236	0,00287	0,00347	0,00420	0,00514	0,00640	0,00824	0,01145
0,90	0,05	0,00080	0,00099	0,00122	0,00149	0,00181	0,00218	0,00264	0,00323	0,00401	0,00515	0,00709
0,92	0,04	0,00045	0,00057	0,00070	0,00085	0,00103	0,00124	0,00150	0,00183	0,00227	0,00291	0,00397
0,94	0,03	0,00022	0,00027	0,00034	0,00041	0,00050	0,00060	0,00073	0,00089	0,00110	0,00140	0,00189
0,96	0,02	0,00008	0,00010	0,00012	0,00015	0,00018	0,00022	0,00026	0,00032	0,00039	0,00050	0,00067
0,98	0,01	0,00001	0,00002	0,00002	0,00003	0,00003	0,00004	0,00005	0,00006	0,00007	0,00009	0,00012
1,00	0,00	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000

Table 13. Function $X_8(\psi, \varphi)$.

$-\cos\psi$	$-\cos\varphi$	−0,2	−0,1	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8
	$\tau_H \diagdown \tau_R$	0,60	0,55	0,50	0,45	0,40	0,35	0,30	0,25	0,20	0,15	0,10
0,80	0,10	2,00203	1,77146	1,54007	1,31056	1,08567	0,86834	0,66187	0,47008	0,29786	0,15222	0,04698
0,82	0,09	1,94395	1,72330	1,50169	1,28166	1,06580	0,85687	0,65790	0,47244	0,30494	0,16163	0,05360
0,84	0,08	1,87521	1,66539	1,45451	1,24493	1,03908	0,83952	0,64906	0,47098	0,30929	0,16952	0,06092
0,86	0,07	1,79412	1,59619	1,39711	1,19909	1,00438	0,81533	0,63455	0,46500	0,31032	0,17540	0,06806
0,88	0,06	1,69837	1,51360	1,32763	1,14249	0,96024	0,78305	0,61328	0,45361	0,30730	0,17867	0,07433
0,90	0,05	1,58474	1,41469	1,24342	1,07277	0,90462	0,74092	0,58378	0,43560	0,29829	0,17858	0,07907
0,92	0,04	1,44839	1,29506	1,14054	0,98647	0,83449	0,68634	0,54389	0,40924	0,28490	0,17409	0,08148
0,94	0,03	1,28135	1,14751	1,01255	0,87788	0,74491	0,61513	0,49014	0,37173	0,26200	0,16364	0,08045
0,96	0,02	1,06843	0,95830	0,84718	0,73621	0,62656	0,51940	0,41604	0,31791	0,22668	0,14447	0,07421
0,98	0,01	0,77131	0,69284	0,61362	0,53445	0,45615	0,37955	0,30555	0,23515	0,16950	0,11006	0,05880
1,00	0,00	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000

Table 14. Function $X_9(\varphi, \psi)$.

$-\cos\psi$	$-\cos\varphi$	−0,2	−0,1	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8
	$\tau_H \diagdown \tau_R$	0,60	0,55	0,50	0,45	0,40	0,35	0,30	0,25	0,20	0,15	0,10
0,80	0,10	0,06534	0,06362	0,06164	0,05938	0,05678	0,05379	0,05032	0,04624	0,04138	0,03535	0,02730
0,82	0,09	0,05039	0,04907	0,04756	0,04583	0,04384	0,04156	0,03891	0,03580	0,03210	0,02754	0,02150
0,84	0,08	0,03767	0,03669	0,03557	0,03429	0,03281	0,03112	0,02916	0,02687	0,02414	0,02078	0,01639
0,86	0,07	0,02707	0,02637	0,02558	0,02466	0,02361	0,02241	0,02101	0,01938	0,01744	0,01507	0,01199
0,88	0,06	0,01848	0,01801	0,01746	0,01685	0,01614	0,01532	0,01438	0,01328	0,01197	0,01038	0,00832
0,90	0,05	0,01175	0,01146	0,01111	0,01072	0,01028	0,00976	0,00917	0,00848	0,00765	0,00666	0,00537
0,92	0,04	0,00675	0,00658	0,00639	0,00616	0,00591	0,00562	0,00528	0,00488	0,00442	0,00385	0,00313
0,94	0,03	0,00330	0,00322	0,00312	0,00302	0,00289	0,00275	0,00259	0,00240	0,00217	0,00190	0,00155
0,96	0,02	0,00120	0,00117	0,00114	0,00110	0,00105	0,00100	0,00094	0,00088	0,00079	0,00070	0,00057
0,98	0,01	0,00021	0,00021	0,00020	0,00020	0,00019	0,00018	0,00017	0,00016	0,00014	0,00012	0,00010
1,00	0,00	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000

Table 15. Function $X_9(\psi, \varphi)$.

$-\cos\psi$	$-\cos\varphi$	—0,2	—0,1	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8
	$\tau_H \backslash \tau_R$	0,60	0,55	0,50	0,45	0,40	0,35	0,30	0,25	0,20	0,15	0,10
0,80	0,10	0,64228	0,56670	0,49251	0,42026	0,35048	0,28379	0,22086	0,16248	0,10970	0,06387	0,02730
0,82	0,09	0,55512	0,49028	0,42662	0,36458	0,30463	0,24729	0,19312	0,14280	0,09716	0,05736	0,02521
0,84	0,08	0,47088	0,41629	0,36268	0,31040	0,25985	0,21147	0,16571	0,12313	0,08442	0,05052	0,02284
0,86	0,07	0,39008	0,34519	0,30109	0,25807	0,21645	0,17657	0,13882	0,10365	0,07160	0,04341	0,02019
0,88	0,06	0,31327	0,27749	0,24232	0,20799	0,17477	0,14291	0,11273	0,08456	0,05884	0,03613	0,01727
0,90	0,05	0,24115	0,21381	0,18693	0,16068	0,13526	0,11086	0,08773	0,06611	0,04632	0,02879	0,01413
0,92	0,04	0,17460	0,15495	0,13562	0,11674	0,09844	0,08088	0,06420	0,04859	0,03428	0,02157	0,01085
0,94	0,03	0,11474	0,10192	0,08931	0,07698	0,06503	0,05355	0,04263	0,03241	0,02302	0,01465	0,00755
0,96	0,02	0,06319	0,05618	0,04928	0,04254	0,03599	0,02970	0,02372	0,01811	0,01295	0,00833	0,00439
0,98	0,01	0,02260	0,02011	0,01766	0,01526	0,01294	0,01070	0,00857	0,00657	0,00473	0,00307	0,00166
1,00	0,00	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000

Table 16. Function $X_{10}(\varphi, \psi) = X_{10}(\psi, \varphi)$.

$-\cos\psi$	$-\cos\varphi$	—0,2	—0,1	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8
	$\tau_H \backslash \tau_R$	0,60	0,55	0,50	0,45	0,40	0,35	0,30	0,25	0,20	0,15	0,10
0,80	0,10	0,02891	0,02536	0,02194	0,01866	0,01554	0,01259	0,00983	0,00729	0,00500	0,00302	0,00140
0,82	0,09	0,02239	0,01966	0,01702	0,01449	0,01208	0,00980	0,00767	0,00571	0,00394	0,00240	0,00113
0,84	0,08	0,01682	0,01477	0,01280	0,01091	0,00911	0,00740	0,00581	0,00434	0,00301	0,00184	0,00089
0,86	0,07	0,01214	0,01067	0,00926	0,00790	0,00660	0,00537	0,00422	0,00316	0,00220	0,00136	0,00067
0,88	0,06	0,00832	0,00732	0,00636	0,00543	0,00454	0,00370	0,00292	0,00219	0,00154	0,00096	0,00048
0,90	0,05	0,00532	0,00468	0,00407	0,00348	0,00291	0,00238	0,00188	0,00142	0,00100	0,00063	0,00032
0,92	0,04	0,00307	0,00270	0,00235	0,00201	0,00169	0,00138	0,00109	0,00083	0,00058	0,00037	0,00019
0,94	0,03	0,00151	0,00133	0,00116	0,00099	0,00083	0,00068	0,00054	0,00041	0,00029	0,00019	0,00010
0,96	0,02	0,00055	0,00049	0,00042	0,00036	0,00031	0,00025	0,00020	0,00015	0,00011	0,00007	0,00004
0,98	0,01	0,00010	0,00009	0,00008	0,00006	0,00005	0,00004	0,00004	0,00003	0,00002	0,00001	0,00001
1,00	0,00	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000

Table 17. Function $X_{12}(\varphi, \psi)$.

$-\cos\psi$	$-\cos\varphi$	—0,2	—0,1	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8
	$\tau_H \backslash \tau_R$	0,60	0,55	0,50	0,45	0,40	0,35	0,30	0,25	0,20	0,15	0,10
0,80	0,10	–0,13701	–0,15928	–0,18629	–0,22016	–0,26434	–0,32488	–0,41343	–0,55545	–0,81888	–1,46659	$-\infty$
0,82	0,09	–0,11595	–0,13468	–0,15733	–0,18565	–0,22244	–0,27258	–0,34534	–0,46056	–0,66939	–1,15428	–3,64024
0,84	0,08	–0,09633	–0,11180	–0,13045	–0,15371	–0,18381	–0,22461	–0,28339	–0,37542	–0,53889	–0,90142	–2,33157
0,86	0,07	–0,07818	–0,09065	–0,10567	–0,12433	–0,14839	–0,18087	–0,22732	–0,29931	–0,42492	–0,69317	–1,60309
0,88	0,06	–0,06152	–0,07128	–0,08300	–0,09753	–0,11620	–0,14128	–0,17693	–0,23165	–0,32565	–0,52013	–1,11555
0,90	0,05	–0,04642	–0,05374	–0,06251	–0,07336	–0,08726	–0,10584	–0,13211	–0,17207	–0,23976	–0,37608	–0,76313
0,92	0,04	–0,03294	–0,03811	–0,04429	–0,05192	–0,06165	–0,07462	–0,09284	–0,12035	–0,16636	–0,25688	–0,49932
0,94	0,03	–0,02123	–0,02454	–0,02849	–0,03336	–0,03955	–0,04777	–0,05926	–0,07648	–0,10495	–0,15984	–0,30015
0,96	0,02	–0,01146	–0,01324	–0,01536	–0,01797	–0,02127	–0,02564	–0,03172	–0,04077	–0,05557	–0,08361	–0,15258
0,98	0,01	–0,00402	–0,00464	–0,00538	–0,00629	–0,00743	–0,00894	–0,01103	–0,01413	–0,01914	–0,02848	–0,05074
1,00	0,00	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000

Table 18. Function $X_{12}(\psi, \varphi)$.

$-\cos\psi$	$-\cos\varphi$	—0,2	—0,1	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8
	$\tau_H \backslash \tau_R$	0,60	0,55	0,50	0,45	0,40	0,35	0,30	0,25	0,20	0,15	0,10
0,80	0,10	4,76197	4,72520	4,61371	4,42312	4,14475	3,76343	3,25263	2,56224	1,58112	–0,03830	$-\infty$
0,82	0,09	5,18049	5,15759	5,05753	4,87652	4,60667	4,23409	3,73473	3,06429	2,12883	0,66238	–3,25866
0,84	0,08	5,66293	5,65496	5,56670	5,39492	5,13243	4,76647	4,27509	3,61867	2,71662	1,35890	–1,51769
0,86	0,07	6,23114	6,23957	6,16366	6,00074	5,74427	5,38244	4,89502	4,24623	3,36584	2,08339	–0,29091
0,88	0,06	6,91914	6,94601	6,88328	6,72881	6,47664	6,11576	5,62734	4,97865	4,10744	2,87186	0,78434
0,90	0,05	7,78344	7,83181	7,78351	7,63695	7,38670	7,02237	6,52622	5,86780	4,99092	3,77442	1,85221
0,92	0,04	8,92706	9,00173	8,96983	8,83037	8,57835	8,20385	7,68980	7,00729	6,10434	4,87436	3,02794
0,94	0,03	10,56435	10,67373	10,66164	10,52775	10,26745	9,87100	9,32148	8,59051	7,62834	6,33742	4,47629
0,96	0,02	13,24844	13,41004	13,42464	13,29275	13,01001	12,56635	11,94408	11,11364	10,02443	8,58132	6,56742
0,98	0,01	19,16871	19,43536	19,4963[illegible]	19,35371	19,00282	18,43183	17,61944	16,53021	15,10467	13,23536	10,69512
1,00	0,00	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞

Table 19. Function $X_{13}(\varphi, \psi)$.

−cos ψ	−cos φ	—0,2	—0,1	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8
	τ_H \ τ_R	0,60	0,55	0,50	0,45	0,40	0,35	0,30	0,25	0,20	0,15	0,10
0,80	0,10	0,20315	0,23403	0,26893	0,30900	0,35592	0,41225	0,48221	0,57346	0,70205	0,91144	1,49220
0,82	0,09	0,17249	0,19862	0,22812	0,26193	0,30144	0,34875	0,40728	0,48315	0,58891	0,75685	1,13512
0,84	0,08	0,14276	0,16547	0,18994	0,21796	0,25063	0,28965	0,33775	0,39974	0,48532	0,61841	0,89235
0,86	0,07	0,11704	0,13465	0,15448	0,17716	0,20355	0,23500	0,27363	0,32316	0,39094	0,49448	0,69520
0,88	0,06	0,09238	0,10624	0,12183	0,13963	0,16031	0,18489	0,21499	0,25340	0,30555	0,38398	0,52943
0,90	0,05	0,06991	0,08036	0,09211	0,10550	0,12104	0,13947	0,16197	0,19055	0,22907	0,28624	0,38861
0,92	0,04	0,04976	0,05718	0,06551	0,07499	0,08598	0,09898	0,11481	0,13483	0,16164	0,20094	0,26936
0,94	0,03	0,03216	0,03694	0,04230	0,04840	0,05545	0,06378	0,07389	0,08663	0,10358	0,12818	0,16999
0,96	0,02	0,01742	0,02000	0,02289	0,02617	0,02997	0,03444	0,03986	0,04666	0,05566	0,06859	0,09013
0,98	0,01	0,00613	0,00703	0,00805	0,00920	0,01052	0,01208	0,01397	0,01633	0,01943	0,02386	0,03110
1,00	0,00	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000

Table 20. Function $X_{13}(\psi, \varphi)$.

−cos ψ	−cos φ	—0,2	—0,1	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8
	τ_H \ τ_R	0,60	0,55	0,50	0,45	0,40	0,35	0,30	0,25	0,20	0,15	0,10
0,80	0,10	5,53595	5,43965	5,30303	5,12427	4,89992	4,62425	4,28812	3,87622	3,36030	2,67608	1,49220
0,82	0,09	5,29349	5,20482	5,07824	4,91218	4,70354	4,44723	4,13512	3,75382	3,27917	2,65904	1,70645
0,84	0,08	5,02968	4,94856	4,83201	4,67867	4,48579	4,24886	3,96069	3,60958	3,17485	2,61382	1,79638
0,86	0,07	4,74095	4,66735	4,56088	4,42038	4,24343	4,02605	3,76192	3,44086	3,04523	2,53987	1,82889
0,88	0,06	4,42245	4,35637	4,26011	4,13267	3,97198	3,77450	3,53478	3,24399	2,88714	2,43531	1,81572
0,90	0,05	4,06718	4,00870	3,92289	3,80891	3,66497	3,48804	3,27341	3,01354	2,69581	2,29652	1,75975
0,92	0,04	3,66448	3,61379	3,53884	3,43895	3,31259	3,15723	2,96886	2,74117	2,46370	2,11726	1,65890
0,94	0,03	3,19646	3,15394	3,09056	3,00579	2,89838	2,76626	2,60611	2,41283	2,17798	1,88643	1,50572
0,96	0,02	2,62848	2,59487	2,54435	2,47651	2,39041	2,28443	2,15604	2,00127	1,81370	1,58202	1,28279
0,98	0,01	1,87166	1,84866	1,81378	1,76677	1,70699	1,63336	1,54418	1,43679	1,30695	1,14729	0,94301
1,00	0,00	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000

Table 21. Function $X_{15}(\varphi, \psi) = X_{14}(\psi, \varphi)$.

−cos ψ	−cos φ	—0,2	—0,1	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8
	τ_H \ τ_R	0,60	0,55	0,50	0,45	0,40	0,35	0,30	0,25	0,20	0,15	0,10
0,80	0,10	0,44532	0,43011	0,41321	0,39445	0,37356	0,35019	0,32382	0,29366	0,25836	0,21523	0,15633
0,82	0,09	0,38187	0,36894	0,35460	0,33867	0,32095	0,30115	0,27884	0,25337	0,22365	0,18758	0,13962
0,84	0,08	0,32140	0,31062	0,29866	0,28539	0,27064	0,25417	0,23564	0,21451	0,18994	0,16029	0,12156
0,86	0,07	0,26417	0,25540	0,24566	0,23486	0,22287	0,20949	0,19445	0,17733	0,15747	0,13364	0,10294
0,88	0,06	0,21052	0,20359	0,19590	0,18738	0,17793	0,16738	0,15554	0,14210	0,12653	0,10793	0,08425
0,90	0,05	0,16081	0,15557	0,14975	0,14330	0,13616	0,12819	0,11926	0,10913	0,09743	0,08350	0,06594
0,92	0,04	0,11554	0,11181	0,10766	0,10308	0,09800	0,09234	0,08599	0,07881	0,07054	0,06072	0,04845
0,94	0,03	0,07535	0,07294	0,07026	0,06730	0,06402	0,06037	0,05628	0,05166	0,04634	0,04006	0,03225
0,96	0,02	0,04118	0,03987	0,03842	0,03682	0,03505	0,03307	0,03086	0,02837	0,02551	0,02213	0,01797
0,98	0,01	0,01462	0,01416	0,01365	0,01308	0,01246	0,01177	0,01099	0,01012	0,00912	0,00794	0,00649
1,00	0,00	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000

Table 22. Function $X_{16}(\varphi, \psi)$.

−cos ψ	−cos φ	—0,2	—0,1	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8
	τ_H \ τ_R	0,60	0,55	0,50	0,45	0,40	0,35	0,30	0,25	0,20	0,15	0,10
0,80	0,10	−0,01070	−0,01241	−0,01447	−0,01703	−0,02033	−0,02481	−0,03122	−0,04121	−0,05882	−0,09758	−0,28170
0,82	0,09	−0,00817	−0,00947	−0,01103	−0,01297	−0,01547	−0,01884	−0,02364	−0,03107	−0,04398	−0,07148	−0,16963
0,84	0,08	−0,00605	−0,00701	−0,00816	−0,00958	−0,01142	−0,01387	−0,01737	−0,02272	−0,03192	−0,05101	−0,11151
0,86	0,07	−0,00431	−0,00499	−0,00580	−0,00681	−0,00810	−0,00983	−0,01227	−0,01599	−0,02231	−0,03513	−0,07272
0,88	0,06	−0,00291	−0,00337	−0,00392	−0,00459	−0,00546	−0,00661	−0,00824	−0,01069	−0,01483	−0,02305	−0,04582
0,90	0,05	−0,00184	−0,00212	−0,00247	−0,00289	−0,00343	−0,00415	−0,00515	−0,00667	−0,00919	−0,01413	−0,02721
0,92	0,04	−0,00104	−0,00121	−0,00140	−0,00164	−0,00195	−0,00235	−0,00291	−0,00376	−0,00515	−0,00784	−0,01471
0,94	0,03	−0,00051	−0,00058	−0,00068	−0,00079	−0,00094	−0,00113	−0,00140	−0,00180	−0,00246	−0,00371	−0,00681
0,96	0,02	−0,00018	−0,00021	−0,00024	−0,00029	−0,00034	−0,00041	−0,00050	−0,00065	−0,00088	−0,00131	−0,00236
0,98	0,01	−0,00003	−0,00004	−0,0000[illegible]	−0,00005	−0,00006	−0,00007	−0,00009	−0,00011	−0,00015	−0,00023	−0,00040
1,00	0,00	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000

Table 23. Function $X_{16}(\psi, \varphi)$.

$-\cos\psi$	$-\cos\varphi$	—0,2	—0,1	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8
	τ_H \ τ_B	0,60	0,55	0,50	0,45	0,40	0,35	0,30	0,25	0,20	0,15	0,10
0,80	0,10	2,66095	2,18598	1,71840	1,26587	0,83669	0,44034	0,08830	−0,20426	−0,41464	−0,50030	−0,28170
0,82	0,09	3,15794	2,64040	2,12899	1,63159	1,15664	0,71367	0,31403	−0,02762	−0,29014	−0,43640	−0,35822
0,84	0,08	3,72944	3,16288	2,60112	2,05232	1,52515	1,02927	0,57602	0,17971	−0,13964	−0,34891	−0,36733
0,86	0,07	4,39888	3,77464	3,15377	2,54481	1,95674	1,39945	0,88442	0,42579	0,04280	−0,23432	−0,34069
0,88	0,06	5,20244	4,50844	3,81622	3,13484	2,47371	1,84312	1,25480	0,72297	0,26641	−0,08663	−0,28137
0,90	0,05	6,19984	5,41826	4,63669	3,86483	3,11275	2,39129	1,71266	1,09143	0,54632	0,10437	−0,18615
0,92	0,04	7,49897	6,60164	5,70217	4,81125	3,93985	3,09966	2,30372	1,56729	0,90955	0,35717	−0,04551
0,94	0,03	9,32261	8,25968	7,19190	6,13141	5,09056	4,08245	3,12145	2,22418	1,41101	0,70934	0,16201
0,96	0,02	12,24069	10,90653	9,56358	8,22648	6,91005	5,62983	4,40272	3,24793	2,18857	1,25473	0,49093
0,98	0,01	18,48897	16,55703	14,60875	12,66454	10,74494	8,87130	7,06660	5,35661	3,77172	2,35036	1,14665
1,00	0,00	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞	∞

Table 24. Function $X_{17}(\varphi, \psi)$.

$-\cos\psi$	$-\cos\varphi$	—0,2	—0,1	0,0	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8
	τ_H \ τ_B	0,60	0,55	0,50	0,45	0,40	0,35	0,30	0,25	0,20	0,15	0,10
0,80	0,10	0,01600	0,01841	0,02113	0,02423	0,02785	0,03216	0,03746	0,04428	0,05364	0,06808	0,09810
0,82	0,09	0,01225	0,01409	0,01616	0,01853	0,02128	0,02456	0,02858	0,03372	0,04074	0,05143	0,07223
0,84	0,08	0,00909	0,01045	0,01199	0,01373	0,01577	0,01818	0,02113	0,02490	0,03002	0,03770	0,05206
0,86	0,07	0,00649	0,00746	0,00855	0,00979	0,01123	0,01294	0,01503	0,01769	0,02127	0,02660	0,03624
0,88	0,06	0,00439	0,00505	0,00579	0,00663	0,00760	0,00875	0,01015	0,01193	0,01432	0,01783	0,02404
0,90	0,05	0,00278	0,00319	0,00365	0,00418	0,00479	0,00551	0,00639	0,00750	0,00899	0,01115	0,01490
0,92	0,04	0,00158	0,00182	0,00208	0,00238	0,00273	0,00314	0,00364	0,00426	0,00509	0,00630	0,00835
0,94	0,03	0,00077	0,00088	0,00101	0,00115	0,00132	0,00152	0,00176	0,00206	0,00246	0,00303	0,00399
0,96	0,02	0,00028	0,00032	0,00037	0,00042	0,00048	0,00055	0,00063	0,00074	0,00088	0,00109	0,00142
0,98	0,01	0,00005	0,00006	0,00006	0,00007	0,00008	0,00010	0,00011	0,00013	0,00015	0,00019	0,00025
1,00	0,00	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000

Table 25. Function $X_{17}(\psi, \varphi)$.

$-\cos\psi$	$-\cos\varphi$	—0,2	—0,1	0,0	0,1	0,2	0.3	0,4	0,5	0,6	0,7	0,8
	τ_H \ τ_B	0,60	0,55	0,50	0,45	0,40	0,35	0,30	0,25	0,20	0,15	0,10
0,80	0,10	4,42426	3,87515	3,33767	2,81594	2,31433	1,83767	1,39149	0,98255	0,61968	0,31598	0,09810
0,82	0,09	4,28874	3,76352	3,24904	2,74918	2,26802	1,81006	1,38043	0,98533	0,63275	0,33426	0,11114
0,84	0,08	4,13042	3,63122	3,14189	2,66604	2,20747	1,77035	1,35940	0,98030	0,64025	0,34947	0,12559
0,86	0,07	3,94562	3,47493	3,01324	2,56389	2,13038	1,71655	1,32672	0,96605	0,64102	0,36062	0,13963
0,88	0,06	3,72938	3,29019	2,85911	2,43921	2,03369	1,64604	1,28019	0,94078	0,63358	0,36648	0,15194
0,90	0,05	3,47475	3,07073	2,67392	2,28709	1,91313	1,55519	1,21678	0,90201	0,61599	0,36554	0,16115
0,92	0,04	3,17125	2,80714	2,44930	2,10020	1,76239	1,43864	1,13204	0,84618	0,58546	0,35572	0,16566
0,94	0,03	2,80163	2,48394	2,17154	1,86654	1,57113	1,28768	1,01882	0,76756	0,53763	0,33384	0,16326
0,96	0,02	2,33295	2,07164	1,81454	1,56335	1,31985	1,08593	0,86370	0,65560	0,46454	0,29432	0,15035
0,98	0,01	1,68199	1,49588	1,31265	1,13352	0,95972	0,79259	0,63357	0,48436	0,34696	0,22395	0,11897
1,00	0,00	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000	0,00000

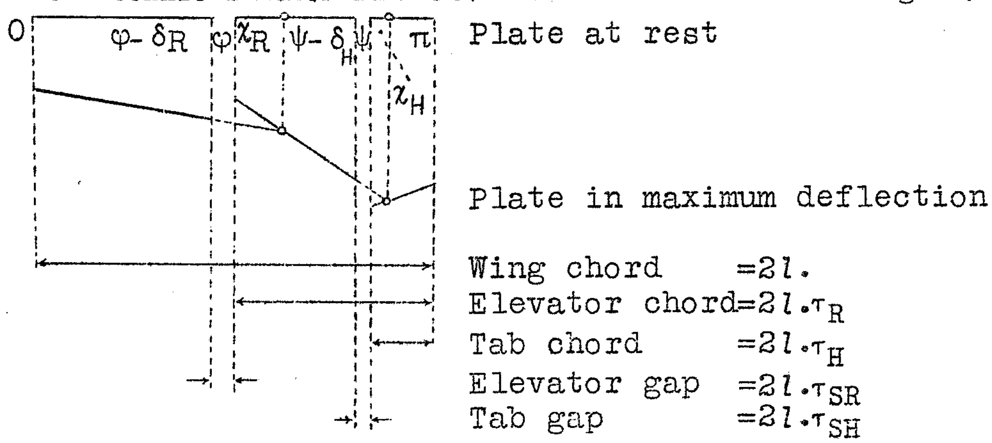


Figure 1.-Geometrical dimensions of the plate.

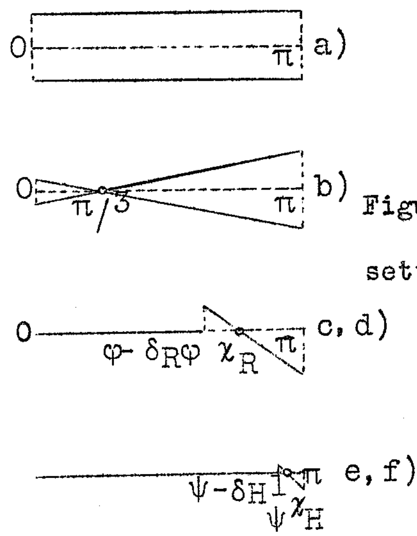
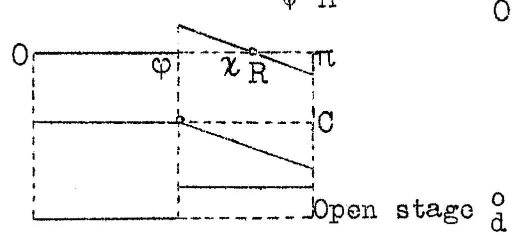


Figure 2.- Wing and elevator oscillations- the mean line is shown in the setting of maximum deflection.



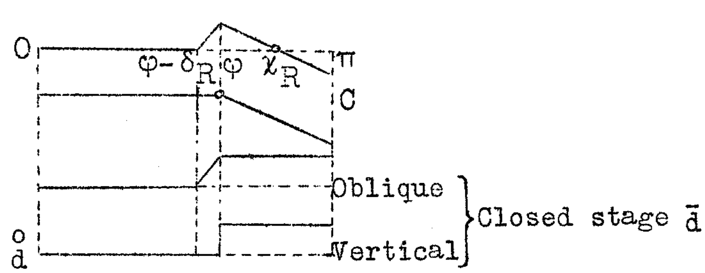


Figure 3a.-If there is a perceptible slot flow, the idealization to slot width 0 is permissible. The mean line of the fin is extended to φ. The gap between fin trailing edge and elevator nose is not bridged over. The relative form change is divided in degrees of freedom c (elevator torsional oscillation) and $\overset{\circ}{d}$ (elevator stage oscillation by open stage).

Figure 3b.-On fin and elevator with setback elevator axis the gap is bridged over by a slope, in the case of vanishing slot flow, the relative form change is divided in the degrees of freedom c (elevator torsional oscillation) and $\bar{d}$ (elevator stage oscillation by closed slanting stage). The transition to slot width 0 is not completely successful (closed vertical stage).

k_a	m_a	$r_a(\varphi)$	$n_a(\varphi)$	$p_a(\psi)$	$q_a(\psi)$
k_b	m_b	$r_b(\varphi)$	$n_b(\varphi)$	$p_b(\psi)$	$q_b(\psi)$
$k_c(\varphi)$	$m_c(\varphi)$	$r_c(\varphi)$	$n_c(\varphi)$	$p_c(\varphi,\psi)$	$q_c(\varphi,\psi)$
$k_d(\varphi)$	$m_d(\varphi)$	$r_d(\varphi)$	$n_d(\varphi)$	$p_d(\varphi,\psi)$	$q_d(\varphi,\psi)$
$k_e(\psi)$	$m_e(\psi)$	$r_e(\varphi,\psi)$	$n_e(\varphi,\psi)$	$p_e(\psi)$	$q_e(\psi)$
$k_f(\psi)$	$m_f(\psi)$	$r_f(\varphi,\psi)$	$n_f(\varphi,\psi)$	$p_f(\psi)$	$q_f(\psi)$

Figure 4.- Tabulation of the force and moment coefficients of the 6 degrees of freedom.

	φ	ψ
φ	φ	φ,ψ
ψ	φ,ψ	ψ

Figure 5.- Relation of the coefficients with φ and ψ.